

## Today's Learning Goal(s):

By the end of the class, I will be able to:

- determine if and where any holes or asymptotes occur for a rational function.
- graph a rational function.

## 2.5 Exploring Graphs of Rational Functions (Holes)

Date: Mar. 4 / 19  
(Every lesson)

HOLES!!!

$$\text{Graph } g(x) = \frac{x^2 + 7x + 12}{x + 3}$$

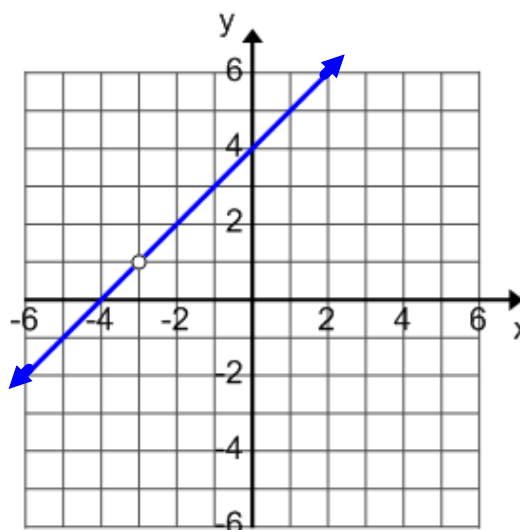
Just like our first unit! Factor first.

$$g(x) = \frac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}}$$

We have the restriction that  $x \neq -3$ , but since we cancel  $(x+3)$  we create a hole in the graph.

So,  $g(x) = (x + 4)$  is a linear function with a hole at  $x = -3$

see [desmos](#) (FIRST)



Ex.1 Graph  $f(x) = \frac{x^2 - 4}{x - 2}$

$$= \frac{(x+2)(x-2)}{x-2}$$

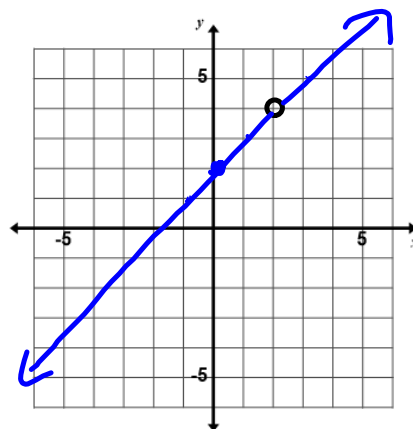
$$= x+2$$

restriction  $x \neq 2$

$$b = 2$$

$$m = 1$$

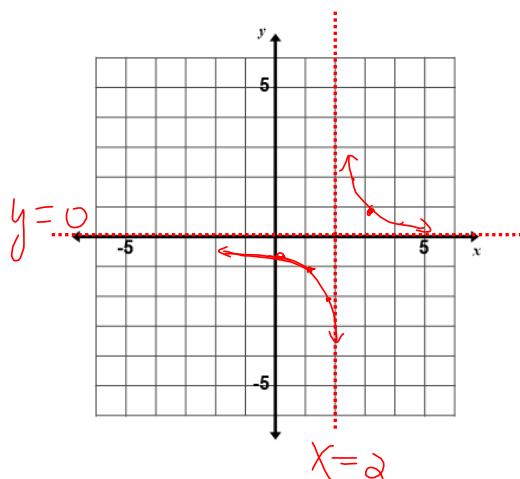
$$= \frac{1}{1} \frac{\text{rise}}{\text{run}}$$



The restriction is that  $x \neq 2$ .  
there is hole at  $x = 2$ .

Ex.2 Graph  $g(x) = \frac{1}{x - 2}$

The restriction is still  $x \neq 2$ .  
there is vertical asymptote at  $x = 2$ .



Summary:

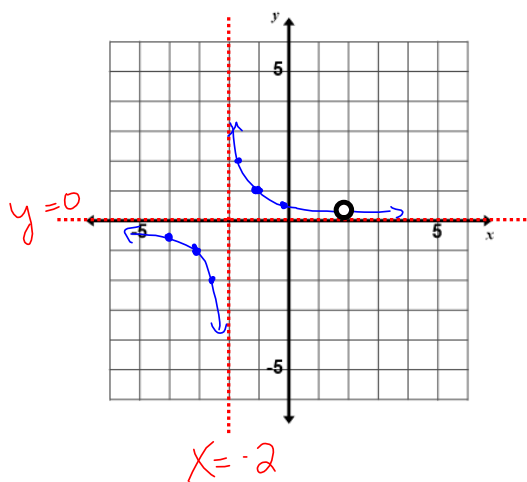
If the restriction divides out, then there is a **hole** at that point.

If the restriction remains, then there is a vertical asymptote at that point.

Ex.3 Graph  $h(x) = \frac{x-2}{x^2-4}$

$$\begin{aligned}
 &= \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} \\
 &= \frac{1}{x+2}
 \end{aligned}$$

hole at  $x=2$   
V.A. at  $x=-2$



Ex.4 As an extra, let's discuss:  $m(x) = \frac{x-2}{x^2+4}$

**You will NOT be expected to graph this.**

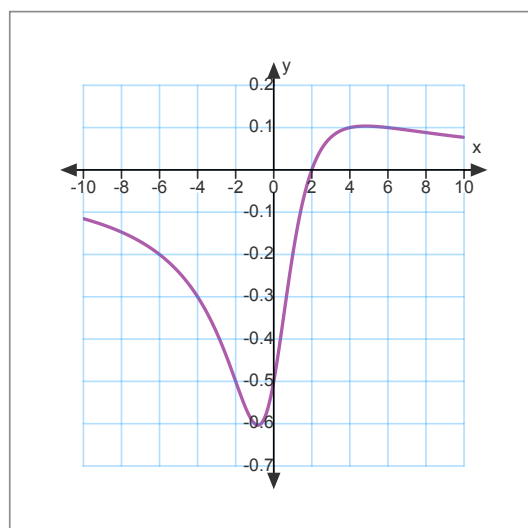
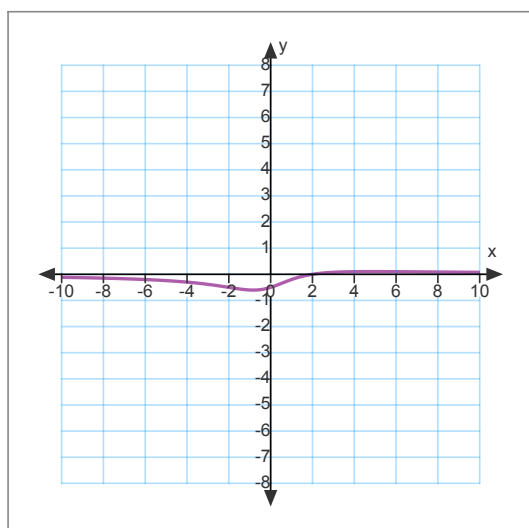
no restrictions,  
no asymptotes

$$\begin{aligned}
 &x^2+4 \neq 0 \\
 &x^2 \neq -4
 \end{aligned}$$

did not reduce/cancel,  
no holes

$$y = \frac{x-2}{x^2+4}$$

$$y = \frac{x-2}{x^2+4}$$



Ex.5 Determine any vertical asymptotes or holes for:

$$f(x) = \frac{x^3 - 4x}{x^3 - x^2 - 6x}$$

$$= \frac{x(x^2 - 4)}{x(x^2 - x - 6)}$$

$$= \frac{\cancel{x}(x-2)(\cancel{x+2})}{\cancel{x}(x-3)(\cancel{x+2})}$$

$$= \frac{x-2}{x-3}$$

Restrictions:  $x \neq 3, -2, 0$

$$f(x) = \frac{x-2}{x-3}$$

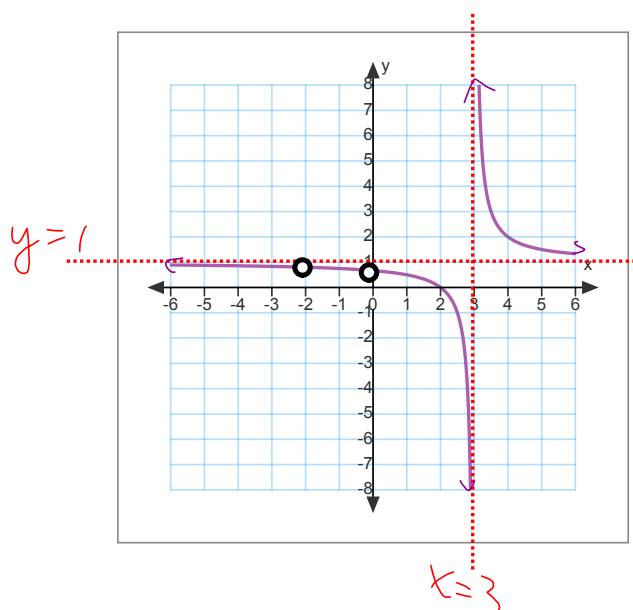
$$x \neq 0, 3, -2$$

Holes at  $x = 0$  and  $x = -2$

(because the  $x$  and  $x+2$  divided out)

vertical asymptote at  $x = 3$

(because the  $x$  and  $x-3$  remained)



$$y = \frac{x^3 - 4x}{x^3 - x^2 - 6x}$$

$$y = \frac{x-2}{x-3}$$

**Are there any Homework Questions you would like to see on the board?**

Last day's work: pp. 70-71 #4def, 5cd, 6a, 7a

Today's Homework Practice includes:  
pp. 70-73 #6bc, 7c, (8,9)ac, 10, 12, 16, 18 [20, 22]  
**+3 Quesons**

**Additional Homework Questions Assigned**

MCR 3UI

### Graphs of Rational Functions

Determine any Vertical Asymptotes or Holes for the following functions.  
Graph each function.

$$a(x) = \frac{x^2 - 2x - 3}{x - 3}$$

$$b(x) = \frac{x^2 + 2x}{x^3 - 4x}$$

$$c(x) = \frac{x^3 - x^2 + 2x - 2}{x - 1}$$

## Final Answer/Graph for Additional Homework Questions Assigned

MCR 3UI

### Graphs of Rational Functions

Determine any Vertical Asymptotes or Holes for the following functions.  
Graph each function.

$$a(x) = \frac{x^2 - 2x - 3}{x - 3}$$

$$b(x) = \frac{x^2 + 2x}{x^3 - 4x}$$

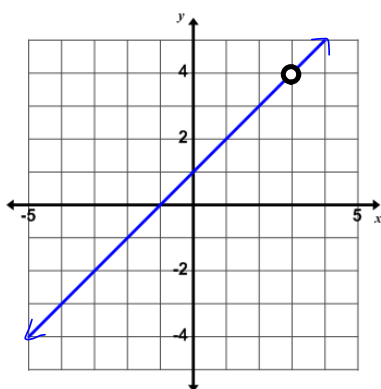
$$c(x) = \frac{x^3 - x^2 + 2x - 2}{x - 1}$$

Simplifies to:

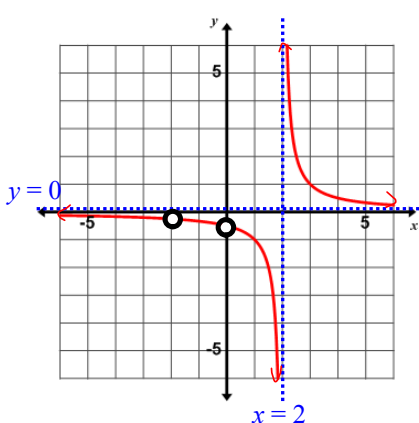
$$a(x) = x + 1$$

$$b(x) = \frac{1}{x - 2}$$

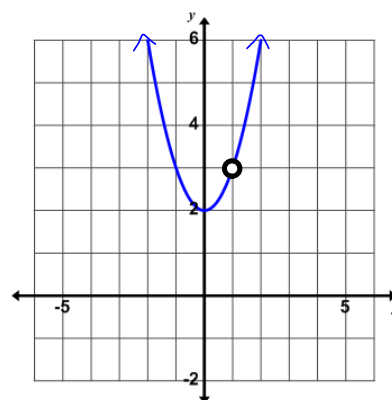
$$c(x) = x^2 + 2$$



The restriction is that  $x \neq 3$ .  
there is hole at  $x = 3$ .



The restrictions are:  $x \neq -2, 0, 2$ .  
there are holes at  $x = -2$  and  $0$ .  
there is vertical asymptote at  $x = 2$ .



The restriction is that  $x \neq 1$ .  
there is hole at  $x = 1$ .