Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 198-199 #1c, 2ac, 3, 4ab, 5–8 [11]
pp. 202-203 #1 – 12, 13 – 17, 19 – 23

9c) $-4\sqrt{3}$
11) $3\sqrt{5}$

3, 11, 15, 16
5, 19

Today's Homework Practice includes:
Review

p. 204 #1 – 9

HIGHLY RECOMMENDED

Worksheet on Class Website:
"Word Problems Involving Quadratics" #1 – 10

p. 198 **2.** Determine the point(s) of intersection algebraically.

a) $f(x) = -x^2 + 6x - 5, g(x) = -4x + 19$
b) $f(x) = 2x^2 - 1, g(x) = 3x + 1$
c) $f(x) = 3x^2 - 2x - 1, g(x) = -x - 6$

a) $-x^2+6x-5 = -4x+19$

$0 = x^2-6x+5-4x+19$

$= x^2-10x+24$

$= (x-6)(x-4)$

$\therefore x=6$ or $x=4$

$g(6) = -4(6)+19$ $\quad$ $g(4) = -4(4)+19$

$= -24+19$ $\quad$ $= -16+19$

$= -5$ $\quad$ $= 3$

$\therefore (6,-5)$ and $(4,3)$ are the solutions

$\rightarrow$ P.O.I s.

p. 198

4. Determine the point(s) of intersection of each pair of functions.

K a) $f(x) = -2x^2 - 5x + 20$, $g(x) = 6x - 1$

$-2x^2 - 5x + 20 = 6x - 1$

$0 = 2x^2 + 5x - 20 + 6x - 1$

$= 2x^2 + 11x - 21$

$= (2x - 3)(x + 7)$

$\therefore x = \frac{3}{2}$ or $x = -7$

$g(-7) = 6(-7) - 1$

$= -42 - 1$

$= -43$

$\therefore (-7, -43)$ and $(\frac{3}{2}, 8)$ are the P.O.I.s

$g(\frac{3}{2}) = \cancel{6}^{3}(\frac{3}{\cancel{2}}) - 1$

$= 9 - 1$

$= 8$

p. 198

11. A quadratic function is defined by $f(x) = 3x^2 + 4x - 2$. A linear function **T** is defined by $g(x) = mx - 5$. What value(s) of the slope of the line would make it a tangent to the parabola?

$\Leftarrow$ if tangent $\Rightarrow b^2 - 4ac = 0$ (1 P.O.I.)

$3x^2 + 4x - 2 = mx - 5$

$3x^2 + 4x - mx - 2 + 5 = 0$

$3x^2 + (4 - m)x + 3 = 0$

$a = 3$ $b = 4 - m$ $c = 3$

$(4 - m)^2 - 4(3)(3) = 0$

$16 - 8m + m^2 - 36 = 0$

$m^2 - 8m - 20 = 0$

$(m - 10)(m + 2) = 0$

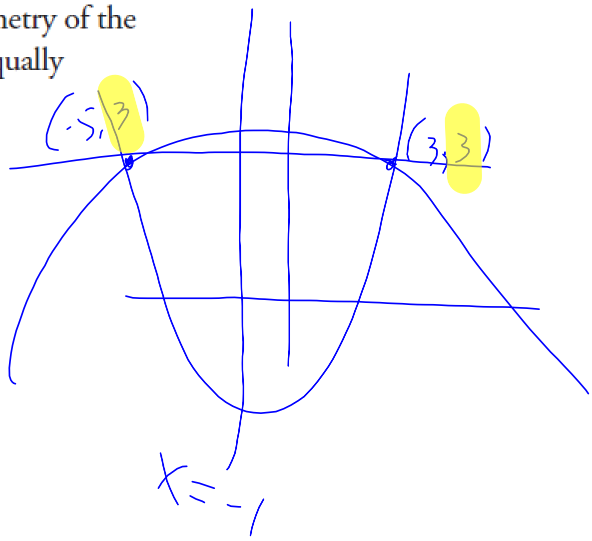
$\therefore m = 10$ or $m = -2$

p. 202 **3.** Determine the equation of the axis of symmetry of the parabola with points $(-5, 3)$ and $(3, 3)$ equally distant from the vertex on either side of it.

AOS: $x = \frac{-5+3}{2}$

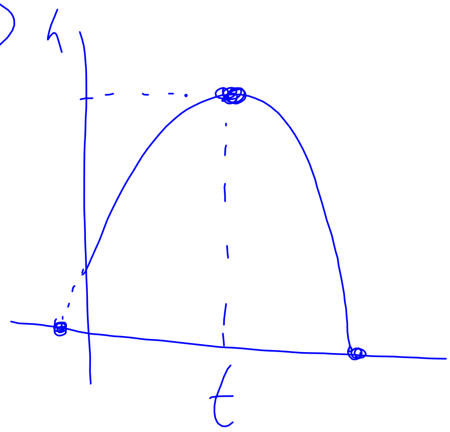
$= \frac{-2}{2}$

$x = -1$



p. 202 **5.** The height, $h(t)$, in metres, of the trajectory of a football is given by $h(t) = 2 + 28t - 4.9t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached.

Find the vertex



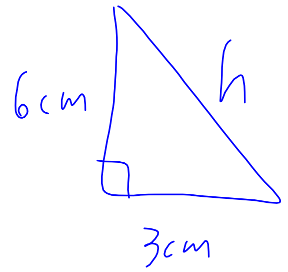
$h(t) = -4.9t^2 + 28t + 2$

$t = \frac{-b}{2a}$

$= \frac{28}{2(-4.9)}$

*See #5 later in file.

p. 202 **11.** What is the perimeter of a right triangle with legs 6 cm and 3 cm? Leave your answer in simplest radical form.



$$P = 6 + 3 + h$$
$$= 9 + h$$
$$= 9 + 3\sqrt{5}$$

$$h^2 = 3^2 + 6^2 \quad (PT)$$
$$= 9 + 36$$
$$= 45$$
$$h = \sqrt{45}$$
$$= \sqrt{9}\sqrt{5}$$
$$= 3\sqrt{5}$$

p. 203 **13.** The population of a Canadian city is modelled by $P(t) = 12t^2 + 800t + 40\ 000$, where t is the time in years. When $t = 0$, the year is 2007.

a) According to the model, what will the population be in 2020?

b) In what year is the population predicted to be 300 000?

a) $t = 2020 - 2007$
$$= 13$$

$$P(13) = 12(13)^2 + 800(13) + 40000$$
$$= 52428$$

b) $P(t) = 300\ 000$

$$\therefore 300000 = 12t^2 + 800t + 40\ 000$$
$$0 = 12t^2 + 800t + 40000 - 300\ 000$$
$$= 12t^2 + 800t - 260\ 000$$

$a = 12 \quad b = 800 \quad c = -260\ 000$

$$t = \frac{-800 \pm \sqrt{800^2 - 4(12)(-260000)}}{2(12)}$$
$$= \frac{-800 \pm \sqrt{13\ 120\ 000}}{24}$$

$t \doteq 117.58$ or $t \doteq -184.2$

$$\doteq 117.5$$

$$\therefore 2007 + 117.5$$
$$\doteq 2124.5$$

p. 203

14. A rectangular field with an area of 8000 m² is enclosed by 400 m of fencing. Determine the dimensions of the field to the nearest tenth of a metre.

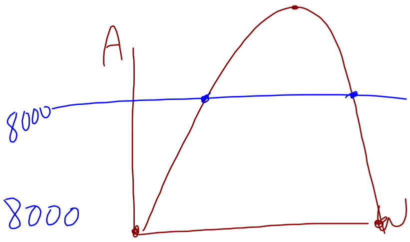
ℓ

w $\boxed{A = 8000\,m^2}$

$400 = 2\ell + 2w$ $\quad A = \ell w$

$400 - 2w = 2\ell$ $\quad 8000 = (200 - w)w$

$200 - w = \ell$ $\quad = 200w - w^2$



$0 = -w^2 + 200w - 8000$

$a = -1 \quad b = 200 \quad c = -8000$

$$w = \frac{-200 \pm \sqrt{200^2 - 4(-1)(-8000)}}{2(-1)}$$

$$= \frac{-200 \pm \sqrt{8000}}{-2}$$

$$\therefore w = \frac{-200 + \sqrt{8000}}{-2} \quad \text{or} \quad w = \frac{-200 - \sqrt{8000}}{-2}$$

$\doteq 55.27 \qquad \doteq 144.72$

$\doteq 55.3 \qquad \doteq 144.7$

$\ell = 200 - w \qquad \ell = 200 - w$

$\doteq 144.7\text{ m} \qquad \doteq 200 - 144.7$

$\doteq 55.3\text{ m}$

OR Using completing the square to solve instead of the Q.R.F. (Quadratic Formula)

$0 = -w^2 + 200w - 8000$

$0 = w^2 - 200w + 8000$

$= w^2 - 200w + 10\,000 - 10000 + 8000$

$= (w - 100)^2 - 2000$

$2000 = (w - 100)^2$

$\pm\sqrt{2000} = w - 100$

$100 \pm \sqrt{2000} = w$

$w \doteq 144.7$ or $w \doteq 55.3$

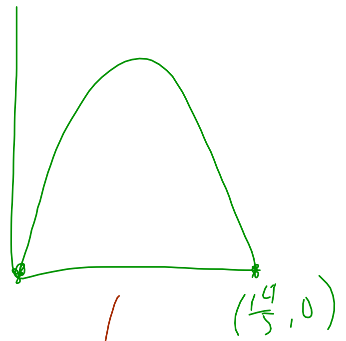
p. 203 **15.** The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t) = 14t - 5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m? Explain.

$h(t) = -5t^2 + 14t$

Let $h(t) = 9$

$9 = -5t^2 + 14t$

$5t^2 - 14t + 9 = 0$



Are there solutions that work for 9 m?
Check using discriminant.

$b^2 - 4ac$

$= (-14)^2 - 4(5)(9)$

$= 196 - 180$

$= 16$

$\therefore$ 2 Real Solutions

$\therefore$ Yes the projectile can reach 9 m.

Can it reach 10 m?

$5t^2 - 14t + 10 = 0$

$b^2 - 4ac$

$= (-14)^2 - 4(5)(10)$

$= 196 - 200$

$= -4$

$\therefore$ No Real Solutions

$\therefore$ the projectile can NOT reach 10 m.

p. 203 **16.** Determine the values of k for which the function $f(x) = 4x^2 - 3x + 2kx + 1$ has two zeros. Check these values in the original equation.

$f(x) = 4x^2 + x(-3+2k) + 1$

$a = 4 \quad b = -3+2k \quad c = 1$

$b^2 - 4ac > 0$

$(-3+2k)^2 - 4(4)(1) > 0$

$9 - 12k + 4k^2 - 16 > 0$

$4k^2 - 12k - 7 > 0$

NOTE: DO NOT WORRY about this type of question, when there is a k^2. You will not be tested on this.

$(2k+1)(2k-7) > 0$

Case 1

$2k+1 > 0$ AND $2k-7 > 0$

$2k > -1 \qquad 2k > 7$

$k > -\frac{1}{2} \qquad k > \frac{7}{2}$

OR $2k+1 < 0$ and $2k-7 < 0$

p. 203 **17.** Determine the break-even points of the profit function $P(x) = -2x^2 + 7x + 8$, where x is the number of dirt bikes produced, in thousands.

Break Even $P(x) = 0$

$0 = -2x^2 + 7x + 8$

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(-2)(8)}}{2(-2)}$

$= \dfrac{-7 \pm \sqrt{49+64}}{-4}$

$= \dfrac{-7 \pm \sqrt{113}}{-4}$

$\rightarrow x \doteq -0.9$ inadmissible or $x \doteq 4.4075$ in thousands

4407.5

$\therefore$ 4408 bikes.

p. 203 **19.** Describe the characteristics that the members of the family of parabolas $f(x) = a(x+3)^2 - 4$ have in common. Which member passes through the point $(-2, 6)$? $\rightarrow V(-3, -4)$

(x, y) $f(x) = a(x+3)^2 - 4$

All members of the family of $f(x)$ above have the vertex (- 3 , - 4).

$6 = a(-2+3)^2 - 4$

$6 = a(1)^2 - 4$

$6+4 = a$

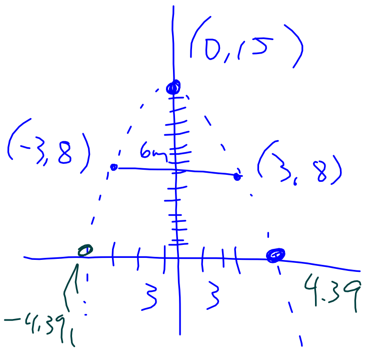
$10 = a$

$\therefore f(x) = 10(x+3)^2 - 4$ is the eq'n.

p. 203 **20.** An engineer is designing a parabolic arch. The arch must be 15 m high, and 6 m wide at a height of 8 m.

a) Determine a quadratic function that satisfies these conditions.

b) What is the width of the arch at its base?



$y = a(x-0)^2 + 15$

$y = ax^2 + 15$

$8 = a(3)^2 + 15$

$8 = 9a + 15$

$8 - 15 = 9a$

$-7 = 9a$

$a = -\frac{7}{9}$

$\therefore y = -\frac{7}{9}x^2 + 15$ is the eq'n

b) Let $y = 0$

$0 = -\frac{7}{9}x^2 + 15$

$-\frac{9}{7}(-15) = \cancel{-\frac{7}{9}}x^2\left(\cancel{-\frac{9}{7}}\right)$

$\frac{135}{7} = x^2$

$x = \pm\sqrt{\frac{135}{7}}$

$\doteq \pm 4.39$

$\therefore$ the width is 8.78 m

p. 203 **21.** Calculate the point(s) of intersection of $f(x) = 2x^2 + 4x - 11$ and $g(x) = -3x + 4$.

$2x^2 + 4x - 11 = -3x + 4$

$2x^2 + 4x + 3x - 11 - 4 = 0$

$2x^2 + 7x - 15 = 0$

$(2x - 3)(x + 5) = 0$

$\therefore x = \frac{3}{2}$ or $x = -5$

$g(x) = -3x + 4$

$\therefore g(\frac{3}{2}) = -3(\frac{3}{2}) + 4$

$= -\frac{9}{2} + \frac{8}{2}$

$= \frac{-1}{2}$

$g(-5) = -3(-5) + 4$

$= 15 + 4$

$= 19$

$\therefore (\frac{3}{2}, \frac{-1}{2})$ and $(-5, 19)$ are the POIs.

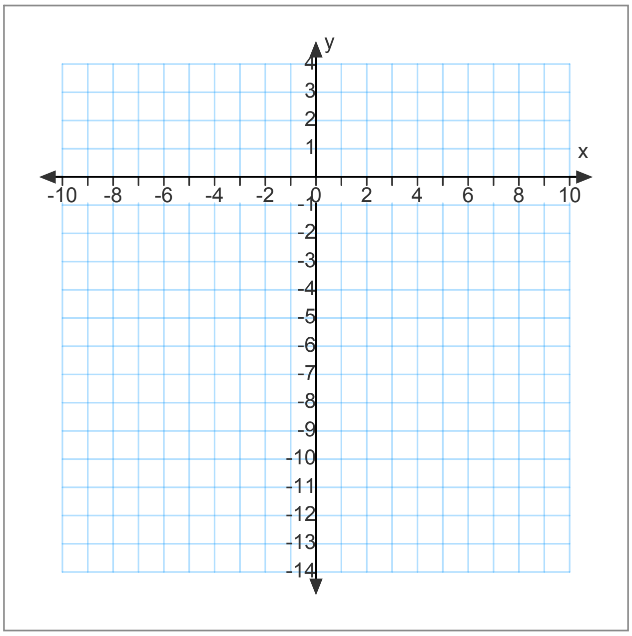
Quadratics Review

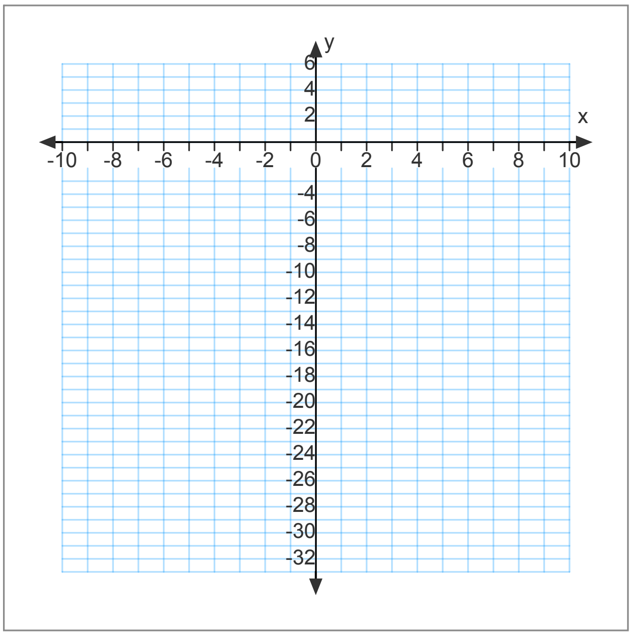
Date: ___________________________

1. For each function below state the direction of the opening, the vertex, axis of symmetry, max or min value, and the domain and range. Finally, sketch the function.

a) $f(x) = -2(x-5)^2 - 4$

b) $f(x) = 2(x-3)(x+5)$





$y = -2(x-5)^2 - 4$

$y = 2(x-3)(x+5)$

2. a) The height, $h(t)$, in metres, of the tragectory of a football is given by $h(t) = 2 + 28t - 4.9t^2$, where t is the time in flight, in seconds. Determine the maximum height of the football and the time when that height is reached.

$h(t) = -4.9t^2 + 28t + 2$

AofS: $t = \frac{-b}{2a}$

$= \frac{-(28)}{2(-4.9)}$

$\doteq 2.857$

$\frac{14}{4.9}$

$= \frac{140}{49}$

$= \frac{20}{7}$

$h(2.86) \doteq -4.9(2.86)^2 + 28(2.86) + 2$

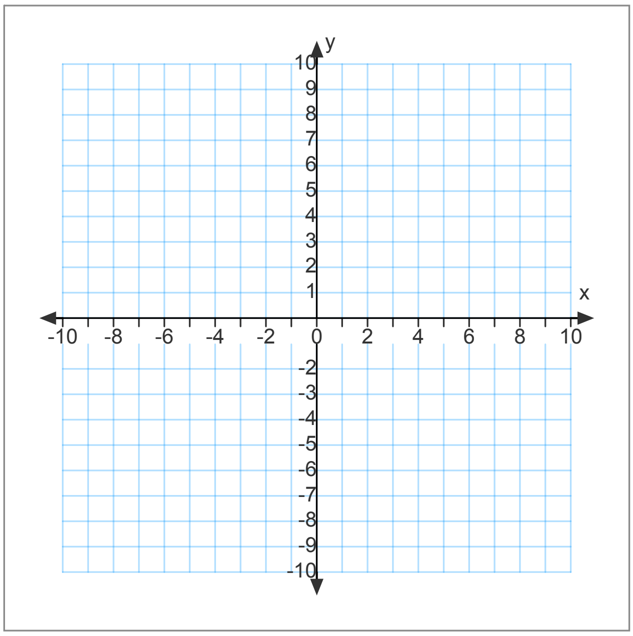
b) How long will it take for the ball to hit the ground?

$h(t) = 0$

$0 = -4.9t^2 + 28t + 2$

3. a) Determine the inverse of $f(x) = -3(x-4)^2 + 2$

b) Graph $f(x)$ and $f^{-1}(x)$



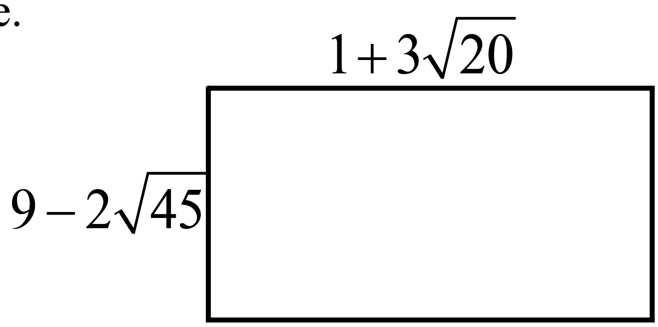
c) Is the inverse a function? Explain using words.

d) State the domain and range of $f(x)$ and $f^{-1}(x)$

4. Express each radical in simplest radical form.

a) $\sqrt{98}$ b) $-5\sqrt{50}$ c) $-2\sqrt{12}+4\sqrt{48}$

5. Determine an expression in lowest terms for the perimeter AND area of the rectangle.



$P = 2l + 2w$

$= 2(1+3\sqrt{20}) + 2(9-2\sqrt{45})$

$= 2 + 6\sqrt{20} + 18 - 4\sqrt{45}$

$= 20 + 6\sqrt{4}\sqrt{5} - 4\sqrt{9}\sqrt{5}$

$= 20 + 6(2)\sqrt{5} - 4(3)\sqrt{5}$

$= 20 + 12\sqrt{5} - 12\sqrt{5}$

$= 20$ units

$A = lw$

$= (1+3\sqrt{20})(9-2\sqrt{45})$

$= (1+3\sqrt{4}\sqrt{5})(9-2\sqrt{9}\sqrt{5})$

$= (1+3(2)\sqrt{5})(9-2(3)\sqrt{5})$

$= (1+6\sqrt{5})(9-6\sqrt{5})$

$-36\sqrt{25}$

$= 9 - 6\sqrt{5} + 54\sqrt{5} - 36(5)$

$= 9 + 48\sqrt{5} - 180$

$= -171 + 48\sqrt{5}$

6. a) The height, $h(t)$, of a projectile, in metres, can be modelled by the equation $h(t)=14t-5t^2$, where t is the time in seconds after the projectile is released. Can the projectile ever reach a height of 9 m?

* if vertex is 9 m or higher. ✓

$$h(t) = -5t^2+14t$$

$$t=\frac{-b}{2a} \quad = -5\left(t^2-\frac{14}{5}t+\left(\frac{7}{5}\right)^2-\frac{7}{5}^2\right)$$

$$=1.4 \quad = -5\left(t-\frac{7}{5}\right)^2-5\left(-\frac{49}{25}\right)$$

$$= -5\left(t-\frac{7}{5}\right)^2+\frac{49}{5}$$

max height is $\frac{49}{5}$

$= 9\frac{4}{5}$ m

∴ Yes.

b) How long will it take for it to hit the ground?

$$h(t)=0$$

$$0=-5t^2+14t$$

$$=-5t\left(t-\frac{14}{5}\right)$$

$$t=0 \quad \text{or} \quad t=\frac{14}{5}$$

$$=2.8 \text{ sec}$$

∴ the ball hits the ground at 2.8 sec.

7. Determine the value(s) for k for which the function has no Real roots.

$$f(x) = 3x^2 - 4x + k$$

$a=3 \quad b=-4 \quad c=k$

$b^2-4ac<0$

$(-4)^2-4(3)(k)<0$

$16-12k<0$

$-12k<-16 \qquad 16<12k$

$k>\frac{-16}{-12} \qquad \frac{16}{12}<k$

$k>\frac{4}{3} \qquad \frac{4}{3}<k$

8. Determine the equation of parabola that has roots $\sqrt{5}$ and $-\sqrt{5}$ and goes through point (-1, 6).

$r=\sqrt{5}$ $s=-\sqrt{5}$

$y=a(x-r)(x-s)$

$y=a(x-\sqrt{5})(x+\sqrt{5})$ ← family

$6=a(-1-\sqrt{5})(-1+\sqrt{5})$

$6=a(1-\sqrt{5}+\sqrt{5}-5)$

$6=a(1-5)$

$6=-4a$

$\frac{6}{-4}=a$

$a=-\frac{3}{2}$

$\therefore y=-\frac{3}{2}(x-\sqrt{5})(x+\sqrt{5})$

is the equation

or $y=-\frac{3}{2}(x^2-5)$

$=-\frac{3}{2}x^2+\frac{15}{2}$

9. Solve $3x^2 - 4x + 2 = 0$

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