Date: ______________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use exponential functions to model exponential growth and decay.

Last day's work: pp. 251-253 #(1,2)ab, 3, 4ab, 5ab, 9

(*Oponal Wkst 4.6 Extra Pracce*)

(text quesons at end of lesson)

Order Change Spring 2018

pp. 251-253 #1 – 5, 9, 10 [12 – 14]

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4.7 Applications Involving Exponential Functions

Date: Apr. 15/19

Ex.1 You invest $1000 at 8% /a compounded annually.
a) Model the amount of money as a growth function.
b) How much will you have after 20 years?

$1000(1.08)^0$ $1000(1.08)^1$ $1000(1.08)^2$ $1000(1.08)^3$

# of years	0	1	2	3			n
Amount	1000	1080					$1000(1.08)^n$

$1000 $1080 $1166.40

Simple Interest

$I = Prt$

$I_1 = 1000(0.08)(1)$
$= 80$

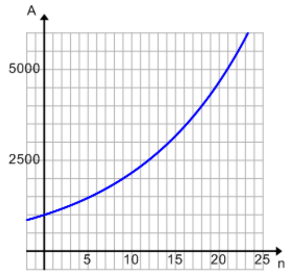
$A_1 = P + I$
$= 1000 + 80$
$= 1080$

$I_2 = 1080(0.08)(1)$
$= 86.40$

$A_2 = 1080 + 86.40$
$= 1166.40$

$A_1 = 1000(1.08)$
$= 1080$

$A_2 = 1080(1.08)$
$= 1166.40$



$A = P + I$
$= P + Prt$
$= P(1+rt)$

$A_1 = 1000 + 80$
$= 1000 + 1000(0.08)(1)$
$= 1000(1 + 0.08)$
$= 1000(1.08)$

$A_2 = 1080(1.08)$
$= \underbrace{1000(1.08)}(1.08)$
$= 1000(1.08)^2$

$A = 1000(1.08)$

Initial Amount 1000

Growth Rate 0.08

Growth Factor 1+0.08
=1.08

$A_n = 1000(1.08)^n$ or $A(n) = 1000(1.08)^n$

Initial Amount
Growth Factor

b) $A(n) = 1000(1.08)^n$
$= 1000(1.08)^{20}$
$\doteq 4660.957$
$\doteq \$4660.96$

b) $4660.96

Ex.2 A superball loses 10% of its height after each bounce.
It was dropped from 12 *m*.
Model the bounce height with a decay function.

Initial Amount 12

Decay Rate 0.1

Decay Factor 1-0.1
= 0.9

$$H = 12(0.90)^n$$

Initial Amount

Decay Factor

$1 \pm r$

Each bounce is 90% of the previous bounce.

The function $f(x) = a(b^x)$ can be used as a model to solve problems involving exponential growth and decay.

$$f(x) = a\,(b^x)$$

Where a is the initial value,
b is the growth factor and
x is the number of compounding periods.

Ex.3 A hockey card is purchased in 1990 for $5.00.
The value increases by 6% each year.
a) Write an equation to represent the growth.
b) Determine the card's value in 2011.

$b = 1 + r$
$= 1 + 0.06$
$= 1.06$

Let V represent the value of the hockey card, in dollars.
Let n represent the number of years since 1990.

a) $V(n) = 5(1.06)^n$

b) $n = 2011 - 1990$
$= 21$

$V(21) = 5(1.06)^{21}$
$\doteq 16.998$
$\doteq \$17.00$

$$V(n) = 5\,(1.06)^n$$

or

$$V(n) = 5\,(1.06)^{n-1990}$$

$17.00

Ex.4 In 1980 the population of the town of St. Albert, Alberta was 20 000. If the town grows at a rate of 2% a year, what was the population in 2014?

Let P represent the population.
Let n represent the number of years since 1980.

$$P(n) = 20000(1.02)^n$$
$$= 20\,000(1.02)^{34}$$

$$n = 2014 - 1980$$
$$= 34$$

$$\doteq 39213.5$$

$$\doteq 39213$$

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There are growth and decay applications that involve doubling times or half-lives. The formula can be altered to:

$$N(t) = N_o(2)^{\frac{t}{d}}$$

t ← total time

d ← doubling time

$$N(t) = N_o\left(\frac{1}{2}\right)^{\frac{t}{d}}$$

t ← total time

d ← amount of time to have **50%** left = **half-life**

t doubles every 3 years

$$N(t) = 8(2)^{\frac{t}{3}}$$

Ex.5 A biology experiment starts with 1000 cells.
After 4 hours the count is estimated to be 256 000.
What is the doubling period for the cells?

Let C represent the number of cells.
Let d represent the doubling period, in hours.

$$C = 1000(2)^{\frac{4}{d}}$$

$$256000 = 1000(2)^{\frac{4}{d}}$$

$$\frac{256000}{1000} = 2^{\frac{4}{d}}$$

$$256 = 2^{\frac{4}{d}}$$

$$2^8 = 2^{\frac{4}{d}}$$

$$\therefore 8 = \frac{4}{d}$$

$$8d = 4$$

$$d = \frac{1}{2}$$

the doubling period for cells is a 1/2 hour.

½ hour

Alternate Method, Same Solution

Ex.5 A biology experiment starts with 1000 cells.
After 4 hours the count is estimated to be 256 000.
What is the doubling period for the cells?

0

$$256000 = 1000(2)^{\frac{4}{d}}$$

$$\frac{256000}{1000} = 2^{\frac{4}{d}}$$

$$256 = 2^{\frac{4}{d}}$$

$$2 \boxed{\wedge} (4 \div 8) = 1.4$$

$$2 \boxed{\wedge} (4 \div 1) = 16$$

$$2 \boxed{\wedge} (4 \div 0.25) = \text{Too big}$$

$$\boxed{2 \boxed{\wedge} (4 \div 0.5) = 256}$$

$\therefore$ my doubling time is $\frac{1}{2}$ an hour.

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 251-253 #(1,2)ab, 3, 4ab, 5ab, 9

(*Oponal Wkst 4.6 Extra Pracce*)

(text quesons on following screens)

Today's Homework Practice includes:

pp. 261-262 # 1 – 8

SWYK NEXT CLASS

1. Each of the following are transformations of $f(x) = 3^x$. Describe each transformation.

   a) $g(x) = 3^x + 3$ c) $g(x) = \frac{1}{3}(3^x)$

   b) $g(x) = 3^{x+3}$ d) $g(x) = 3^{\frac{x}{3}}$

2. For each transformation, state the base function and then describe the transformations in the order they could be applied.

   a) $f(x) = -3(4^{x+1})$ c) $h(x) = 7(0.5^{x-4}) - 1$

   b) $g(x) = 2\left(\frac{1}{2}\right)^{2x} + 3$ d) $k(x) = 5^{3x-6}$

3. State the y-intercept, the equation of the asymptote, and the domain and range for each of the functions in questions 1 and 2.

PRACTISING

4. Each of the following are transformations of $h(x) = (\frac{1}{2})^x$. Use words to describe the sequence of transformations in each case.

   a) $g(x) = -\left(\frac{1}{2}\right)^{2x}$

   b) $g(x) = 5\left(\frac{1}{2}\right)^{-(x-3)}$

   c) $g(x) = -4\left(\frac{1}{2}\right)^{3x+9} - 6$

if $g(x)=\frac{1}{x}$

$f(x)=\frac{a}{k(x-d)}+c$

5. Let $f(x) = 4^x$. For each function that follows,

K
   - state the transformations that must be applied to $f(x)$
   - state the y-intercept and the equation of the asymptote
   - sketch the new function
   - state the domain and range

   a) $g(x) = 0.5f(-x) + 2$ c) $g(x) = -2f(2x - 6)$

   b) $h(x) = -f(0.25x + 1) - 1$ d) $h(x) = f(-0.5x + 1)$

$f(x)=af(k(x-d))+c$

$=ab^{k(x-d)}+c$

$k=0.25$

$=\frac{1}{4}$

$g(x)=a(4^{k(x-d)})+c$

$=0.5(4^{-1(x-0)})+2$

$=0.5(4^{-x})+2$

$=0.5\left(\frac{1}{4}\right)^x+2$

Asymptote: $y=2$

y-int, let $x=0$

$g(0)=0.5\left(\frac{1}{4}\right)^0+2$

$=0.5(1)+2$

$=2.5$

$D_g: \{x \in \mathbb{R}\}$

$R_g: \{y \in \mathbb{R} \mid y>2\}$

b) $h(x)=-(4^{0.25x+1})-1$

$=-(4^{0.25(x+4)})-1$

Asymptote: $y=-1$

y-int, let $x=0$

$h(0)=-(4^{0.25(0)+1})-1$

$=-(4^1)-1$

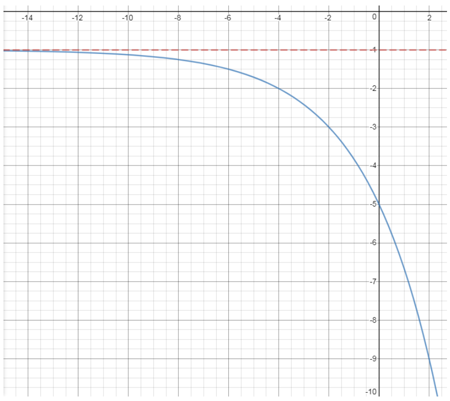
$=-4-1$

$=-5$

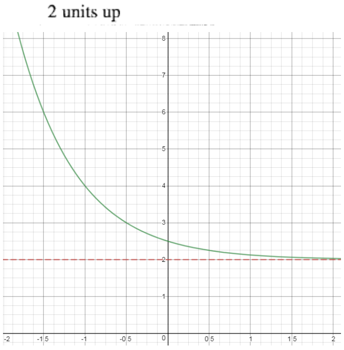
$D_h: \{x \in \mathbb{R}\}$

$R_h: \{y \in \mathbb{R} \mid y<-1\}$

- reflection in the x-axis
- horizontal stretch of 4
- translation 1 down and 4 left



- vertical compression by a factor of $\frac{1}{2}$
- reflection in the y-axis
- translation of 2 units up



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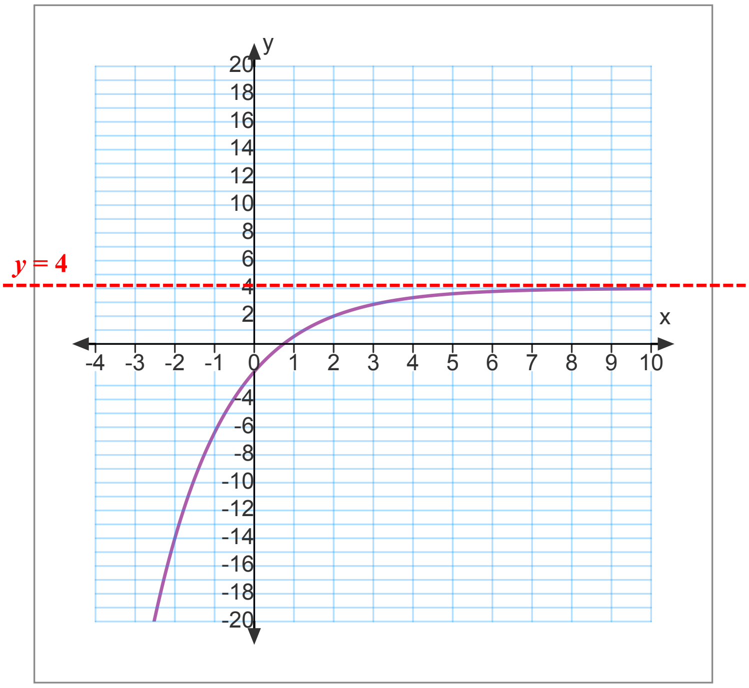
13. Use your knowledge of transformations to sketch the function $g(x) = 4 - 2\left(\frac{1}{3}\right)^{-0.5x+1}$.

$= -2\left(\frac{1}{3}\right)^{-0.5(x-2)} + 4$

$= -2\left(3^{-1}\right)^{-0.5(x-2)} + 4$

$= -2(3)^{0.5(x-2)} + 4$



$$y = -2(3)^{-0.5(x-2)} + 4$$