# 5.6 The Sine Law (Spring 2019)-s19e-p — April 30, 2019

Last day's work: pp. 300-301 #6 – 9ace, 10, 12 [15] b   6c

Review p. 304 #1 – 13

3, 4, 5,

p. 300

**6.** Angle $\theta$ is a principal angle that lies in quadrant 2 such that $0° \le \theta \le 360°$.
**K** Given each trigonometric ratio,

i) determine the exact values of $x$, $y$, and $r$

ii) sketch angle $\theta$ in standard position

iii) determine the principal angle $\theta$ and the related acute angle $\beta$ to the nearest degree

a) $\sin\theta = \frac{1}{3}$  d) $\csc\theta = 2.5$

b) $\cot\theta = -\frac{4}{3}$  e) $\tan\theta = -1.1$

c) $\cos\theta = -\frac{1}{4}$  CXR  f) $\sec\theta = -3.5$

i) S  $x = -1$  $r = 4$

$\therefore y^2 = 4^2 - (-1)^2$
$= 15$
$\therefore y = \sqrt{15}$ (Q II)

ii)


iii) $\cos\theta = -\frac{1}{4}$

$\theta = \cos^{-1}(-\frac{1}{4})$
$\doteq 104.4$
$\doteq 104°$
$\therefore \beta \doteq 180° - 104°$
$= 76°$

p. 301 **10.** Given each point $P(x, y)$ lying on the terminal arm of angle $\theta$,

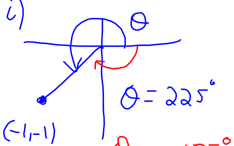
i) state the value of $\theta$, using both a counterclockwise and a clockwise rotation

ii) determine the primary trigonometric ratios

a) $P(-1, -1)$  c) $P(-1, 0)$

b) $P(0, -1)$  d) $P(1, 0)$

a) i)




$\theta = 225°$

$\theta = -135°$

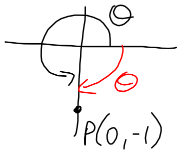
ii) $r^2 = (-1)^2 + (-1)^2$
$= 1+1$
$\therefore r = \sqrt{2}$

$\sin\theta = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

$\tan\theta = \frac{-1}{-1}$
$= 1$

b) i)




$\theta = 270°$

$\theta = -90°$

ii) $r^2 = 0^2 + (-1)^2$
$r = 1$

SYR CXR TYX

$\sin\theta = \frac{-1}{1}$
$= -1$

$\cos\theta = \frac{0}{1}$
$= 0$

$\tan\theta = \frac{-1}{0}$
$=$ undefined

p. 301 **Extending**

**15.** Given angle $\theta$, where $0° \le \theta \le 360°$, solve for $\theta$ to the nearest degree.

a) $\cos 2\theta = 0.6420$

b) $\sin(\theta + 20°) = 0.2045$

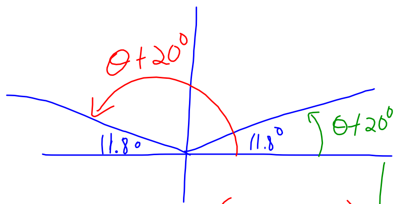
c) $\tan(90° - 2\theta) = 1.6443$

b) let $\beta = \theta + 20°$

$\therefore \sin\beta = 0.2045$

$\beta = \sin^{-1}(0.2045)$

$\doteq 11.800$




$\therefore \theta + 20° \doteq (180° - 11.8)$

$\theta + 20° \doteq 168.2°$

$\theta \doteq 168.2° - 20°$

$\doteq 148.2°$

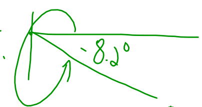
$\therefore \theta + 20° \doteq 11.8°$

$\theta \doteq 11.8° - 20°$

$\doteq -8.2°$

but $0° \le \theta \le 360°$

$\therefore$



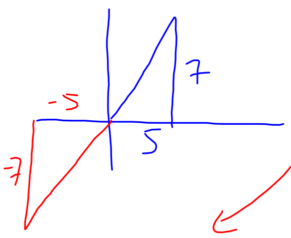

$\therefore \theta \doteq 351.8°$

p. 304 **3.** A trigonometric ratio is $\frac{7}{5}$. What ratio could it be, and what angle might it be referring to?

$\tan\theta = \frac{7}{5}$

$\theta \doteq 54.46$

$\doteq 54.5^\circ$




or $\theta \doteq 234.5^\circ$

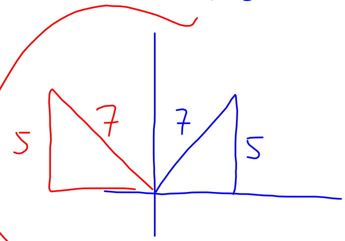
$\csc\theta = \frac{7}{5}$

$\frac{1}{\sin\theta} = \frac{7}{5}$

$\sin\theta = \frac{5}{7}$

$\theta \doteq 45.58$

$\doteq 45.6^\circ$




or $\theta \doteq 134.4^\circ$

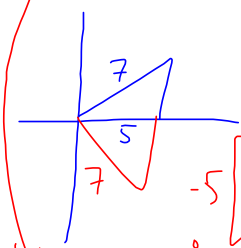
$\sec\theta = \frac{7}{5}$

$\frac{1}{\cos\theta} = \frac{7}{5}$

$\cos\theta = \frac{5}{7}$

$\theta \doteq 44.41$

$\doteq 44.4^\circ$




or $\theta \doteq 315.6^\circ$

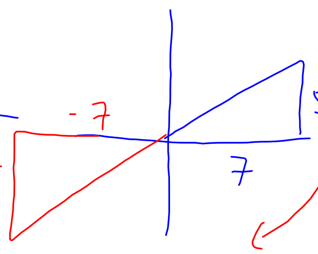
$\cot\theta = \frac{7}{5}$

$\frac{1}{\tan\theta} = \frac{7}{5}$

$\tan\theta = \frac{5}{7}$

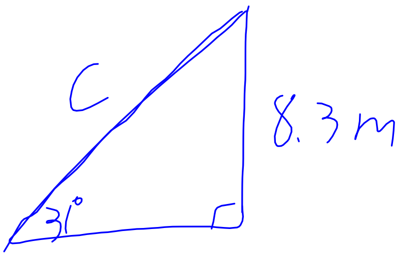
$\theta \doteq 35.53$

$\doteq 35.5^\circ$




or $\theta \doteq 215.5^\circ$

p. 304 **4.** Claire is attaching a rope to the top of the mast of her sailboat so that she can lower the sail to the ground to do some repairs. The mast is 8.3 m long, and with her eyes level with the base of the mast, the top forms an angle of 31° with the ground. How much rope does Claire need if 0.5 m of rope is required to tie to the mast? Round your answer to the nearest tenth of a metre.




Let $t$ represent the total length of rope needed, in m.

$t = c + 0.5$

$\doteq 16.11 + 0.5$

$\doteq 16.61$

$\doteq 16.6$ m

Find $c$

$\sin 31^\circ = \frac{8.3}{c}$

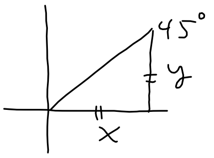
$\therefore c = \frac{8.3}{\sin 31^\circ}$

$\doteq 16.11$

p. 304 **5.** If $\csc\theta < \sec\theta$ and $\theta$ is acute, what do you know about $\theta$?

$0^\circ < \theta < 90^\circ$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$




at 45° $y = x$

if $\csc\theta < \sec\theta$

$\therefore \frac{r}{y} < \frac{r}{x}$

$\frac{r}{y} - \frac{r}{x} < 0$

$\frac{rx - ry}{xy} < 0$

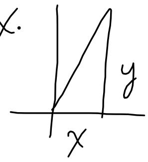
$\frac{r(x-y)}{xy} < 0$

denominator $\therefore x \neq 0, y \neq 0$

numerator

$x - y < 0$

$x < y$

ex. 


$\therefore 45^\circ < \theta < 90^\circ$

p. 304 **13.** Angle $\theta$ lies in quadrant 2. Without using a calculator, which ratios must be false? Justify your reasoning.

**a)** $\cos\theta = 2.3151$
**b)** $\tan\theta = 2.3151$
**c)** $\sec\theta = 2.3151$
**d)** $\csc\theta = 2.3151$
**e)** $\cot\theta = 2.3151$
**f)** $\sin\theta = 2.3151$

a) False: $0 \le \cos\theta \le 1$ *

b) False: tan is −ve in QII

c) False: sec $\left(\frac{1}{\cos}\right)$ is −ve in QII

d) True: csc → $\frac{1}{\sin}$ is +ve in QII

e) False: cot → $\frac{1}{\tan}$ is −ve in QII

f) False: $0 \le \sin\theta \le 1$ *

For a & f, ∵ hypotenuse is the denominator the resulting fraction/decimal must be between 0 and 1.

# Today's Learning Goal(s):

By the end of the class, I will be able to:

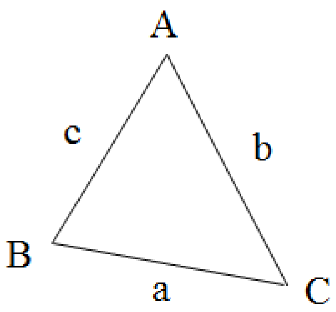
a) use the Sine Law to solve a triangle that is the **ambiguous** case.

## 5.6 The Sine Law

Date: Apr. 30/19

Recall: We use the Sine Law when we are have an "opposite pair".
The formula:

The Sine Law can be used with any triangle, even if it is not a right triangle.
Given any triangle



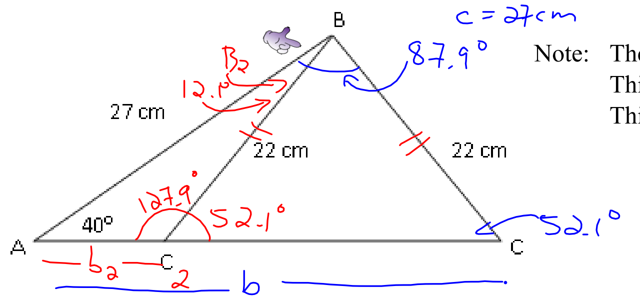

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**or**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex. 1 Consider $\Delta ABC$, $\angle A = 40°$, AB=27 cm, and BC = 22 cm. ***Make a sketch.***

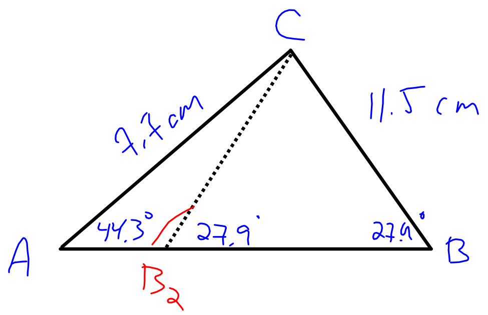
$c = 27\text{ cm}$ $a = 22\text{ cm}$

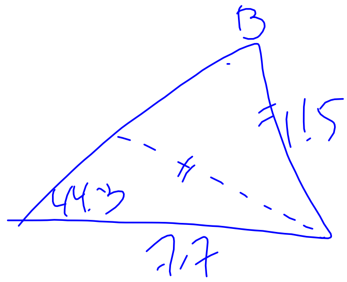



Note: There are 2 different ways to sketch $\Delta ABC$ using this information. This means there are two possible ways to solve this triangle. This is the ambiguous case of the Sine Law.

| $\angle C$ | $\angle B$ | $b$ |
|---|---|---|
| $\frac{\sin C}{27} = \frac{\sin 40°}{22}$ <br> $22 \sin C = 27 \sin 40°$ <br> $\sin C = \frac{27 \sin 40°}{22}$ <br> $C = \sin^{-1}\left(\frac{27 \sin 40°}{22}\right)$ <br> $\doteq 52.08$ <br> $\doteq 52.1°$ | $B \doteq 180° - 40° - 52.1°$ <br> $\doteq 87.9°$ | $\frac{b}{\sin 87.9} \doteq \frac{22}{\sin 40°}$ <br> $b \doteq \sin 87.9 \times \frac{22}{\sin 40°}$ <br> $\doteq 34.202$ <br> $\doteq 34.20\text{ cm}$ <br> ∠C= 52.1°,∠B= 87.9°, b=34.2 cm |
| $\angle C_2$ | $\angle B_2$ | $b_2$ |
| $C_2 \doteq 180° - 52.1$ <br> $= 127.9°$ | $B_2 \doteq 180° - 40° - 127.9°$ <br> $\doteq 12.1°$ | $\frac{b_2}{\sin 12.1°} \doteq \frac{22}{\sin 40°}$ <br> $b_2 \doteq \sin 12.1° \times \frac{22}{\sin 40°}$ <br> $\doteq 7.174$ <br> $\doteq 7.17\text{ cm}$ <br> ∠C= 127.9°,∠B= 12.1°, b=7.2 cm |

Ex. 2 Solve $\Delta ABC$, $\angle A = 44.3°$, a = 11.5 cm, and b = 7.7 cm. ***Make a sketch.***

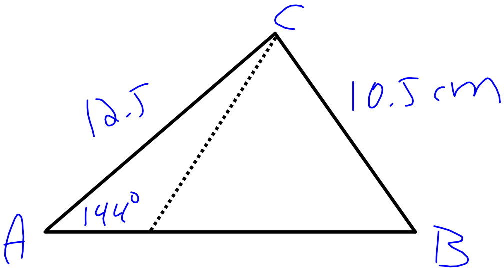


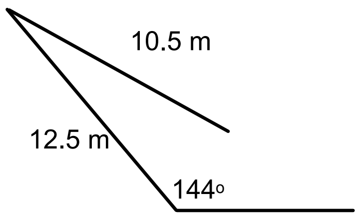




| $\angle B$ | $\angle C$ | c |
|---|---|---|
| $\frac{\sin B}{7.7} = \frac{\sin 44.3°}{11.5}$<br>$B = \sin^{-1}\left(\frac{7.7 \sin 44.3°}{11.5}\right)$<br>$\doteq 27.88$<br>$\doteq 27.9°$ | $C \doteq 180° - 44.3° - 27.9°$<br>$\doteq 107.8°$ | $\frac{c}{\sin 107.8°} \doteq \frac{11.5}{\sin 44.3°}$<br>$c \doteq \sin 107.8 \times \frac{11.5}{\sin 44.3°}$<br>$\doteq 15.677$<br>$\doteq 15.68$ cm<br>∠B= 27.9°,∠C= 107.8°, c=15.7 cm |

| $\angle B_2$ | $\angle C_2$ | |
|---|---|---|
| $B_2 \doteq 180° - 27.9°$<br>$\doteq 152.1°$ | $C_2 \doteq 180° - 44.3° - 152.1°$<br>$\doteq -16.4°$<br>∴ only 1 triangle is possible.<br>∠B= 152.1°,∠C= -16.4° | |

Ex. 3 Solve $\triangle ABC$, $\angle A = 144^\circ$, a = 10.5 cm, and b = 12.5 cm. ***Make a sketch.***



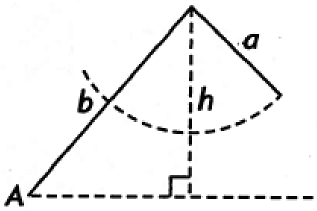

| $\angle B$ | $\angle C$ | |
|---|---|---|
| $\frac{\sin B}{12.5} = \frac{\sin 144^\circ}{10.5}$<br>$B = \sin^{-1}\left(12.5 \times \frac{\sin 144}{10.5}\right)$<br>$\doteq 44.4^\circ$ | $C \doteq 180^\circ - 144 - 44.4$<br>$= -8.4^\circ$<br>$\therefore$ NO triangles are possible<br>$\angle B = 44.4^\circ, \angle C = -8.4^\circ$ | **See Next Slide for New Summaries** |

The ambiguous case arises in a SSA (side, side, angle) triangle. In this situation, depending on the size of the given angle and the lengths of the given sides, the sine law calculation may lead to 0, 1, or 2 solutions.

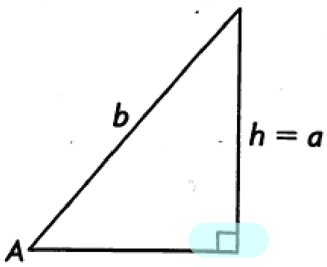
## Need to Know

- In the ambiguous case, if $\angle A$, $a$, and $b$ are given and $\angle A$ is acute, there are four cases to consider. In each case, the height of the triangle is $h = b \sin A$.

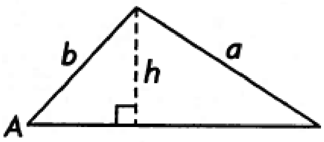
If $\angle A$ is acute and $a < h$, no triangle exists.



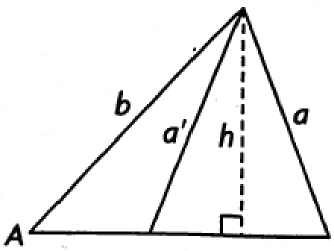

If $\angle A$ is acute and $a = h$, one right triangle exists.




If $\angle A$ is acute and $a > b$, one triangle exists.

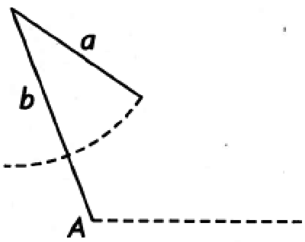



If $\angle A$ is acute and $h < a < b$, two triangles exist.

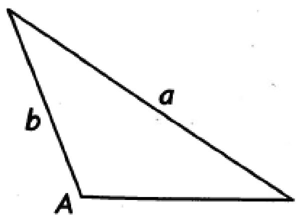



If $\angle A$, $a$, and $b$ are given and $\angle A$ is obtuse, there are two cases to consider.

If $\angle A$ is obtuse and $a < b$ or $a = b$, no triangle exists.




If $\angle A$ is obtuse and $a > b$, one triangle exists.



**Are there any Homework Questions you would like to see on the board?**

Last day's work: pp. 300-301 #6 – 9ace, 10, 12 [15]
Review p. 304 #1 – 13

Today's Homework Practice includes:

pp. 318-319 #1, 2, 3a, 4, 5ac, 7 [15,17]

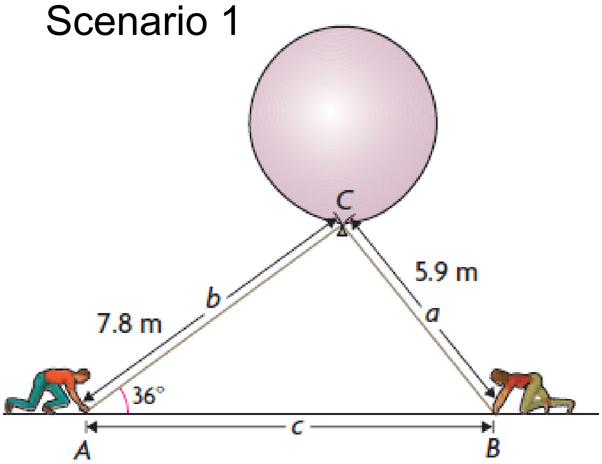
Ex. 4
Albert and Belle are part of a scientific team studying thunderclouds.
The team is about to launch a weather balloon into an active part of the cloud.
Albert's rope is 7.8 m long and makes an angle of $36^0$ with the ground.
Belle's rope is 5.9 m long.
How far, to the nearest tenth of a metre, is Albert from Belle?

Sketch a diagram... where is Belle located?

This is called an **ambiguous** case. We don't have enough information to determine exactly where Belle is located.

So.... we have to give both scenarios and give all possible answers.

Scenario 1




Scenario 2