Date: ______________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) prove trigonometric identities.

#11 spotlights
↳ $h \doteq 9.4\,m$

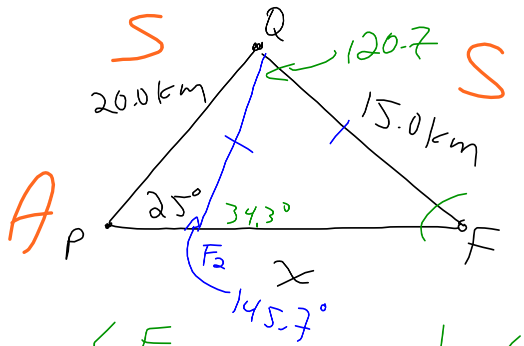
Last day's work: pp. 338-339 #1 – 5, 8 – ~~13~~
p. 340 #2

p. 339 #13 ***See 2 soluons that follow.***

#9 → ambiguous

9. Two forest fire stations, P and Q, are 20.0 km apart. A ranger at station Q sees a fire 15.0 km away. If the angle between the line PQ and the line from P to the fire is $25°$, how far, to the nearest tenth of a kilometre, is station P from the fire?

Be Careful if SSA.



$\angle F$

$$\frac{\sin F}{20} = \frac{\sin 25°}{15}$$

$$F = \sin^{-1}\left(20 \times \frac{\sin 25°}{15}\right)$$

$\doteq 34.29$

$\doteq 34.3°$

$\angle Q$

$Q \doteq 180° - 25° - 34.3°$

$\doteq 120.7°$

x

$$\frac{x}{\sin 120.7°} \doteq \frac{15}{\sin 25°}$$

$$x \doteq \sin 120.7° \times \frac{15}{\sin 25°}$$

$\doteq 30.51$

$\doteq 30.5$ km

$\angle F_2$

$F \doteq 180° - 34.3°$

$\doteq 145.7°$

$\angle Q_2$

$Q_2 \doteq 180° - 25° - 145.7°$

$\doteq 9.3°$

x

$$\frac{x}{\sin 9.3°} \doteq \frac{15}{\sin 25°}$$

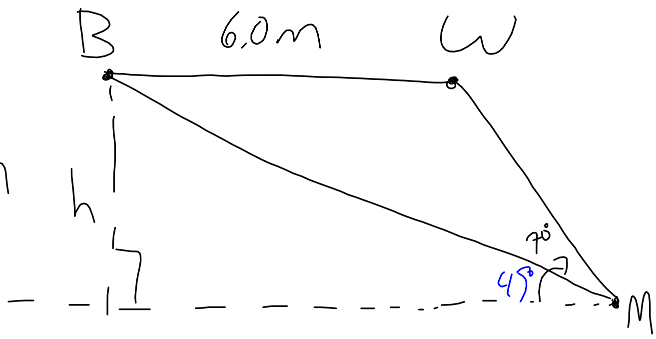
$$x \doteq \sin 9.3° \times \frac{15}{\sin 25°}$$

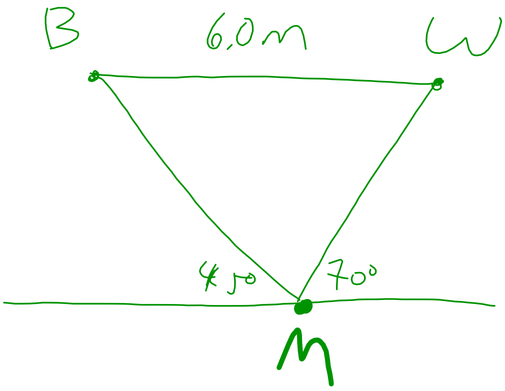
$\doteq 5.73$

$\doteq 5.7$ km

11. Two spotlights, one blue and the other white, are placed 6.0 m apart on a track on the ceiling of a ballroom. A stationary observer standing on the ballroom floor notices that the angle of elevation is 45° to the blue spotlight and 70° to the white one. How high, to the nearest tenth of a metre, is the ceiling of the ballroom?

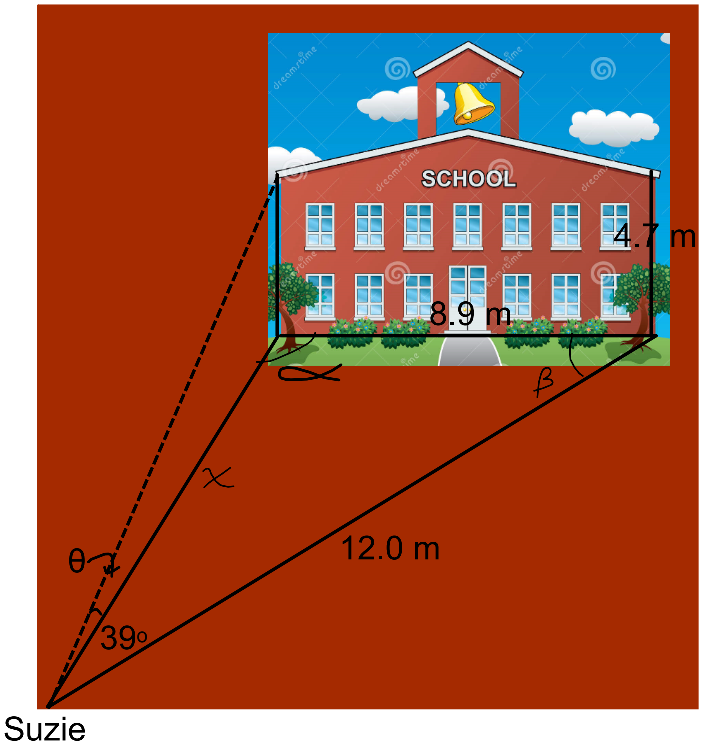
$h = 9.4\,m$





p. 339 #13 **Soluon #1**

13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?



$$\frac{\sin\alpha}{12} = \frac{\sin 39^\circ}{8.9}$$

$$\alpha = \sin^{-1}\left(12 \times \frac{\sin 39^\circ}{8.9}\right)$$

$$\doteq 58.05^\circ$$

But α is obtuse

$\therefore \alpha \doteq 121.9^\circ$

$\therefore \beta \doteq 180^\circ - 39^\circ - 121.9^\circ$

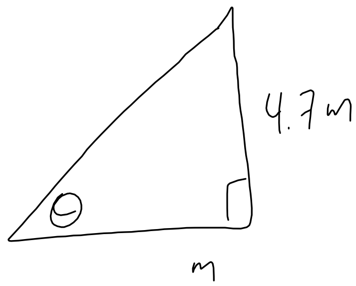
$\doteq 19.1^\circ$

$$\frac{x}{\sin 19.1^\circ} = \frac{8.9}{\sin 39^\circ}$$

$$x \doteq \sin 121.9^\circ \times \frac{8.9}{\sin 39^\circ}$$

$\doteq 4.62$

$\doteq 4.6$



$$\tan\theta \doteq \frac{4.7}{4.6}$$

$$\theta \doteq \tan^{-1}\left(\frac{4.7}{4.6}\right)$$

$\doteq 45.6$

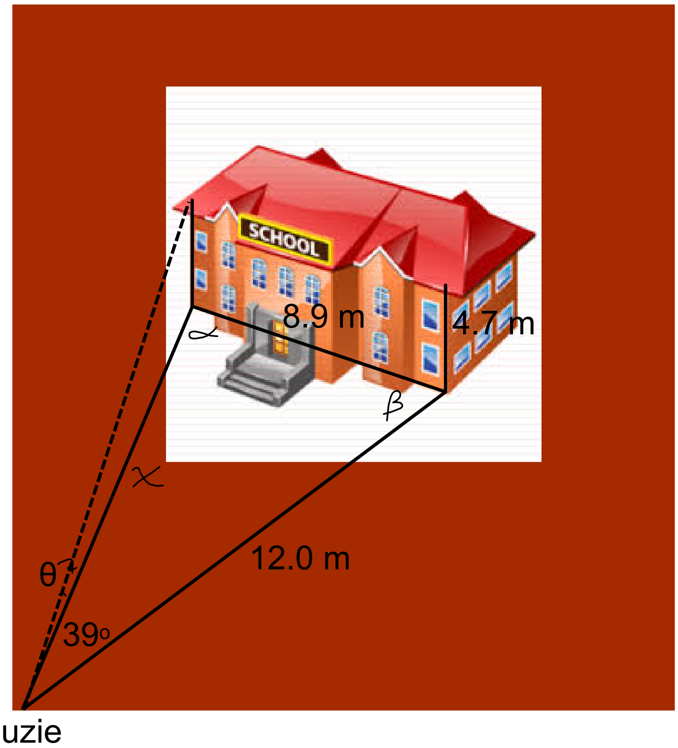
$\doteq 46^\circ$

$\therefore$ the angle of elevation is 46° [some texts have 46° at back.]

but see next page.

p. 339 #13 **Soluon #2**

13. While standing at the left corner of the schoolyard in front of her school, Suzie estimates that the front face is 8.9 m wide and 4.7 m high. From her position, Suzie is 12.0 m from the base of the right exterior wall. She determines that the left and right exterior walls appear to be 39° apart. From her position, what is the angle of elevation, to the nearest degree, to the top of the left exterior wall?



$$\frac{\sin\alpha}{12} = \frac{\sin 39^\circ}{8.9}$$

$$\alpha = \sin^{-1}\left(12 \times \frac{\sin 39^\circ}{8.9}\right)$$

$$\doteq 58.05^\circ$$

$$\therefore \beta \doteq 180^\circ - 39^\circ - 58.1^\circ$$

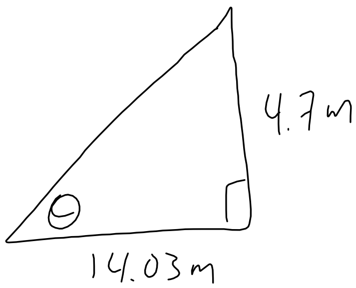
$$\doteq 82.9^\circ$$

$$\frac{x}{\sin 82.9^\circ} \doteq \frac{8.9}{\sin 39^\circ}$$

$$x \doteq \sin 82.9^\circ \times \frac{8.9}{\sin 39^\circ}$$

$$\doteq 14.033$$

$$\doteq 14.03$$



$$\tan\theta \doteq \frac{4.7}{14.03}$$

$$\theta \doteq \tan^{-1}\left(\frac{4.7}{14.03}\right)$$

$$\doteq 18.51$$

$$\doteq 19^\circ$$

$\therefore$ the angle of elevation is 19°

[Some texts say 18° at the back?]

5.5 Trigonometric Identities

Date: May 6/19

Equations are valid for only certain values of the variable.
For example:

$2x + 1 = 7$

$2x = 7 - 1$

$2x = 6$

$x = 3$

$x = 3$ is the only value to make the equation true.

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$\therefore x = 7$ or $x = -2$

$x = 7$ and $x = -2$ are the only values to make the equation true.

Identities are valid for **all values** of the variable.
For example:

$2(x + 3) = 2x + 6$ $\quad$ $x^2 + 6x + 9 = (x + 3)^2$

Let's start with the circle definitions to develop some identities that we can use later.

SYR CXR TYX

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

To Prove an Identity:

*** Separate the LS and RS, and work on them separately**

Ex.1 Prove that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$LS = \tan\theta$

$= \dfrac{y}{x}$

$RS = \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\frac{y}{r}}{\frac{x}{r}}$

$= \dfrac{y}{r} \div \dfrac{x}{r}$

$= \dfrac{y}{\cancel{r}} \times \dfrac{\cancel{r}}{x}$

$= \dfrac{y}{x}$

$\therefore LS = RS$

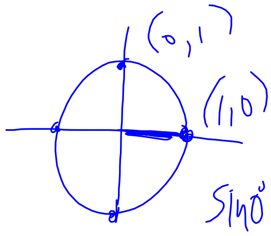
$\therefore$ QED

Restriction

$\cos\theta \neq 0$

$\theta \neq \cos^{-1}(0)$

$\theta \neq 90°, 270°$



$\cos 0° = \dfrac{1}{1}$

$= 1$

$\sin 0° = \dfrac{0}{1}$

$= 0$

Q.E.D. (also written QED)

"quod erat demonstrandum"

"that which was to be demonstrated"

Ex.2 Prove that $\sin^2\theta + \cos^2\theta = 1$

$LS = \sin^2\theta + \cos^2\theta$ $\quad\quad$ $RS = 1$

$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{y^2 + x^2}{r^2}$ $\quad\quad$ $x^2 + y^2 = r^2$ (PT)

$= \frac{r^2}{r^2}$

$= 1$

$\therefore LS = RS$

$\therefore$ QED

Ex.3 Prove that

Use "known" identities; i.e. known since Ex.1&2

a) $\dfrac{\cos\theta\tan\theta}{\sin\theta}=1$

$LS=\dfrac{\cos\theta\tan\theta}{\sin\theta}$ $RS=1$

$=\dfrac{\cancel{\cos\theta}\left(\dfrac{\sin\theta}{\cancel{\cos\theta}}\right)}{\sin\theta}$

$=\dfrac{\sin\theta}{\sin\theta}$

$=1$

$\therefore LS=RS$

$\therefore$ QED

$\sin^2\theta+\cos^2\theta=1$

$\cos^2\theta=1-\sin^2\theta$

$\sin^2\theta=1-\cos^2\theta$

b) $\cos\theta=\dfrac{1}{\cos\theta}-\sin\theta\tan\theta$

$LS=\cos\theta$ $RS=\dfrac{1}{\cos\theta}-\sin\theta\tan\theta$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin^2\theta}{\cos\theta}$

$=\dfrac{1-\sin^2\theta}{\cos\theta}$

$=\dfrac{\cos^2\theta}{\cos\theta}$ $\dfrac{A^2}{A}$

$=\cos\theta$

$\therefore LS=RS$

$\therefore$ QED

Identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 338-339 #1 – 5, 8 – 13
p. 340 #2

Study for the Unit 5 Summative!

Today's Homework Practice includes:
p. 310 #1 – 6

Work ahead? pp. 310-311 #8, 10 – 12 [14]
Worksheet a – j (***online***)

Note: Sometimes using substitution can help simplify a question.
Ex. Simplify (1 – cosθ)(1 + cosθ) Change to (1 – *a*)(1 + *a*)

$= 1 + \cos\theta - \cos\theta - \cos^2\theta$ $= 1 - a^2$

$= 1 - \cos^2\theta$

$= \sin^2\theta$