Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the sum of the terms of a geometric series.

Last day's work: pp. 452-453 #(1 – 7)ace, 11, 13 [15,16]

p. 452 **4.** **i)** Determine whether each series is arithmetic.

ii) If the series is arithmetic, calculate the sum of the first 25 terms.

a) $-5 + 1 + 7 + 13 + \ldots$

b) $2 + 10 + 50 + 250 + \ldots$

c) $1 + 1 + 2 + 3 + \ldots$

d) $18 + 22 + 26 + 30 + \ldots$

e) $31 + 22 + 13 + 4 + \ldots$

f) $1 - 3 + 5 - 7 + \ldots$

a) $d_1 = 1-(-5) = 6$ $d_2 = 7-1 = 6$

$\therefore$ a. series

$a = -5$

$d = 6$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{25} = \frac{25}{2}[2(-5) + (24)(6)]$$

$$= \frac{25}{2}[-10 + 144]$$

$$= \frac{25}{2}[134]$$

$$= 25[67]$$

$$= 1675$$

p. 453

13. Sara is training to run a marathon. The first week she runs 5 km each day. The next week, she runs 7 km each day. During each successive week, each day she runs 2 km farther than she ran the days of the previous week. If she runs for five days each week, what total distance will Sara run in a 10 week training session?

W1 W2 W3

5×5 7×5 9×5

$= 25$ $= 35$ $= 45$

$\therefore 25 + 35 + 45 + \ldots$

a. series

$a = 25$

$d = 10$

$n = 10$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(25) + 9(10)]$$

$$= 5[50 + 90]$$

$$= 5[140]$$

$$= 700 \text{ km}$$

Review:

Sequence: An ordered set of numbers separated by commas .
Each individual number is called a TERM.
The terms are t_1 , t_2 , t_3 , t_4 , ... , t_n

Arithmetic Sequence: has a common ***difference*** between the terms.
In an arithmetic sequence, the first term is ***a*** and the common difference is ***d***

$t_1 = a$, and the terms are $a, a+d, a+2d, a+3d, \ldots$

The general term is $t_n = a + (n - 1)d$

Geometric Sequence: has a common ***ratio*** between the terms.

In a geometric sequence, the first term is ***a*** and the common **ratio** is ***r***

$t_1 = a$, and the terms are $a, ar, ar^2, ar^3, \ldots$

The general term is $t_n = ar^{n-1}$

Series: An ordered set of numbers separated by addition and/or subtraction signs .
Each individual number is called a TERM.
But now we have: $t_1 + t_2 + t_3 + t_4 + \ldots + t_n$

$S_1 = t_1 \; (= a)$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$

$S_n = t_1 + t_2 + t_3 + t_4 + \ldots + t_n$

An **Arithmetic Series** is the **sum** of the terms of an arithmetic sequence.

The Arithmetic Series Formula:

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \textbf{or} \quad S_n = \frac{n}{2}[t_1 + t_n]$$

where n is the term's position number,
$a = t_1$,
d is the common difference, and
t_n = last term

7.6 Geometric Series

Date: May 30/19

A **Geometric Series** is the **sum** of the terms of a geometric sequence.

Ex.1 Your grandma agrees to give you 1 cent on Dec.1, 2 cents on Dec.2, 4 cents on December 3, etc. until Christmas Day.
What will be your total sum?

$2^{24} \cdot 2^{1}$

$t_n = ar^{n-1}$ $\qquad t_{25} = (1)(2)^{25-1}$

$\qquad = 2^{24}$

$$S_{25} = 1 + 2 + 4 + 8 + \ldots + 2^{23} + \overset{t_{25}}{2^{24}} \quad ①$$

② (multiply by "r") $2S_{25} = 2 + 4 + 8 + \ldots + 2^{23} + 2^{24} + 2^{25}$

③ (subtract up)

④ $2S_{25} - S_{25} = 2^{25} - 1$

⑤ $S_{25} = 2^{25} - 1$

$= 33554431$ cents ⑥

$= \$335\ 544.31$

Thank you grandma!

In general:

$$\begin{aligned}
S_n &= a + ar + ar^2 + \dots + ar^{n-3} + ar^{n-2} + \overset{t_n}{ar^{n-1}} \\
rS_n &= \cancel{ar} + \cancel{ar^2} + \dots + \cancel{ar^{n-3}} + \cancel{ar^{n-2}} + \cancel{ar^{n-1}} + ar^n \\
S_n &= a + \cancel{ar} + \cancel{ar^2} + \dots + \cancel{ar^{n-3}} + \cancel{ar^{n-2}} + \cancel{ar^{n-1}}
\end{aligned}$$

(subtract)

$$rS_n - S_n = ar^n - a$$

(factor both sides)

$$(r-1)S_n = a(r^n - 1)$$

The Geometric Series Formula:

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

where n is the term's position number,
$a = t_1$, and
r is the common ratio

Ex.2 Find S_{10} for 5 + 15 + 45 + ...

g.series

$r_1 = \frac{15}{5} = 3$ $r_2 = \frac{45}{15} = 3$

$a = 5$

$r = 3$

$n = 10$

$t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{10} = \frac{5(3^{10} - 1)}{3 - 1}$

$= \frac{5(3^{10} - 1)}{2}$

$= 147\ 620$

$S_{10} = 147\ 620$

Ex.3 Find the sum of: 3 - 6 + 12 - 24 + . . . + 768

$3 + (-6) + (12) + (-24) + \ldots + 768$

g.series

$a = 3$

$r = -2$

$r_1 = \frac{-6}{3} = -2$ $r_2 = \frac{12}{-6} = -2$

$n = ?$

We need to find n (first)

$t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$t_n = 768$

$768 = ar^{n-1}$

$768 = 3(-2)^{n-1}$

$\frac{768}{3} = (-2)^{n-1}$

$256 = (-2)^{n-1}$

try

$256 = 2^y$

$256 = 2^8$

$\therefore y = 8$

$y = n - 1$

$8 = n - 1$

$n = 9$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{3((-2)^n - 1)}{-2 - 1}$

$S_9 = \frac{3((-2)^9 - 1)}{-3}$

$= 513$

$n = 9$ $S_9 = 513$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 452-453 #(1 – 7)ace, 11, 13 [15,16]

Today's Homework Practice includes:

pp. 459-461 #(1 – 6)ace, 9, 11, 13 [16,18]