Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the "future value" of an annuity earning compound interest.

Last day's work: pp. 498-499 #3 – 6, 8, 9, 11

4, 5

p. 498 **4.** Chandra borrows some money at 7.2%/a compounded annually. After 5 years, she repays $12 033.52 for the principal and the interest. How much money did Chandra borrow?

4. $A = 12033.52$ $\quad 12033.52 = P(1+0.072)^5$

$P = ?$ $\quad P = \dfrac{12033.52}{(1.072)^5}$

$i = \dfrac{0.072}{1}$ $\quad \doteq 8499.996$

$n = 5 \times 1$ $\quad \doteq \$8500$

$= 5$

p. 498 **5.** Nazir saved $900 to buy a plasma TV. He borrowed the rest at an interest rate of 18%/a compounded monthly. Two years later, he paid $1429.50 for the principal and the interest. How much did the TV originally cost?

5. $A = 1429.50$ $\quad 1429.50 = P\left(1+\frac{0.18}{12}\right)^{24}$

$P = ?$ $\quad P = \dfrac{1429.50}{\left(1+\frac{0.18}{12}\right)^{24}}$ $\quad$ Original Cost

$i = \dfrac{0.18}{12}$ $\quad \doteq \$999.998$ $\quad = \$1000 + \$900$

$n = 2 \times 12$ $\quad \doteq \$1000$ $\quad = \$1900$

$= 24$

p. 499 **11.** Steve wants to have \$25 000 in 25 years. He can get only 3.2%/a interest compounded quarterly. His bank will guarantee the rate for either 5 years or 8 years.

- In 5 years, he will probably get 4%/a compounded quarterly for the remainder of the term.
- In 8 years, he will probably get 5%/a compounded quarterly for the remainder of the term.

a) Which guarantee should Steve choose, the 5-year one or the 8-year one?

b) How much does he need to invest?

11.a) choose the 8-yr guarantee

$A = 25\,000$

$P = ?$

$i = \frac{0.05}{4}$

$n = 17 \times 4$
$= 68$

$P = \frac{25\,000}{\left(1 + \frac{0.05}{4}\right)^{68}}$

$\doteq 10\,741.819$

$\doteq \$10\,741.82$

$P \doteq \frac{10\,741.82}{\left(1 + \frac{0.032}{4}\right)^{32}}$

$\doteq 8\,324.168$

$\doteq \$8\,324.17$

$\therefore$ Steve needs to invest \$8 324.17

If chose 5 year

$P_1 = \frac{25\,000}{\left(1 + \frac{0.04}{4}\right)^{80}}$

$= 11\,277.95$

$P_2 = \frac{11\,277.95}{\left(1 + \frac{0.032}{4}\right)^{20}}$

$= \$9\,616.56$

Steve would have to invest 9616.56 in the 5 yr. guarantee, which is \$1292.39 more.

8.4 Annuities: Future Value

Date: June 7/19

Annuity: an investment with regular deposits or withdrawals.

The **future value** of an annuity is the **sum** of all the regular payments **AND** interest earned.

Note: A **simple** annuity is an annuity in which the payments coincide with the compounding period.

An **ordinary** annuity is an annuity in which the payments are made at the end of each interval.

***Unless otherwise stated,* each annuity in this chapter is a simple, ordinary annuity.**

Ex.1 You quit smoking a pack a day "cold turkey".
You save the money for cigarettes and deposit it athe end of each half year in an account earning 6% /a compounded semi-annually.
Determine the future value of this annuity in 20 years.

① 1 pack a day @ \$10/pack

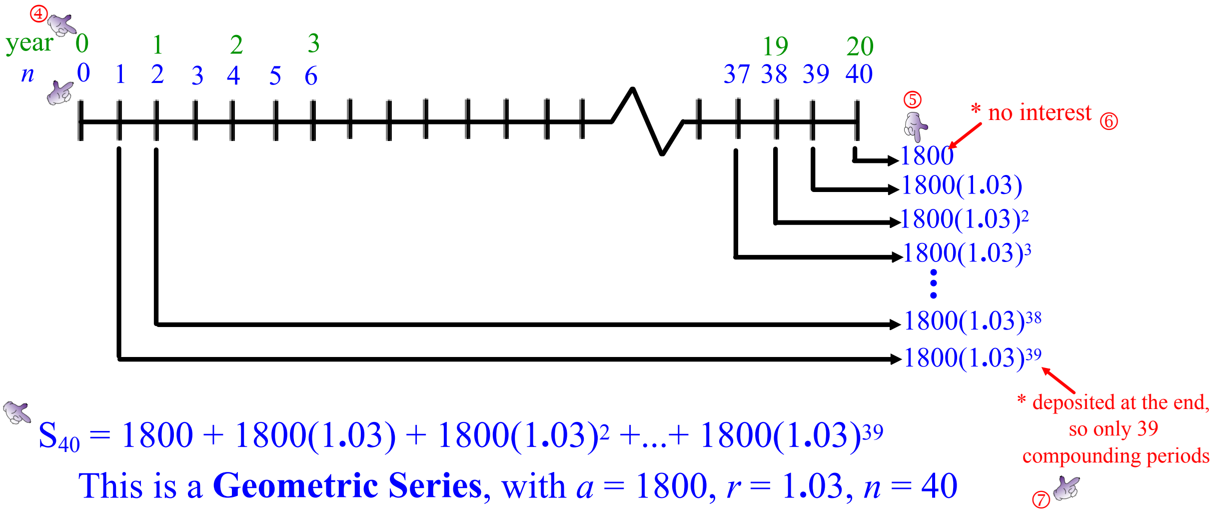
Each deposit = \$10 x 30 x 6

② = \$1800

$A = P(1 + i)^n$

③ $i = \frac{0.06}{2}$
$= 0.03$

$n = 20 \times 2$
$= 40$



$S_{40} = 1800 + 1800(1.03) + 1800(1.03)^2 + ... + 1800(1.03)^{39}$

This is a **Geometric Series**, with $a = 1800$, $r = 1.03$, $n = 40$

Note: $r = 1 + i$

Use $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{40} = \frac{1800(1.03^{40} - 1)}{1.03 - 1}$

$\doteq \$135\ 722.27$

you would have \$135 722.27 in 20 years.

Discuss Interest earned?

\$1800 x 40
= \$ 72 000
\$63 722.27

Making a formula:

Let R represent the regular payment.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{R((1+i)^n - 1)}{(1+i) - 1}$$

$$\boxed{FV = \frac{R((1+i)^n - 1)}{i}}$$

where R is the regular payment
i is the interest rate per compound period
n is the number of compound periods

Read pp. 507-508 Example 2 (both solutions)
Read the Key Ideas/Need to Know p.510

Today's Homework Practice includes:

pp. 511-512 #2, 5ac, 6, 7