

Date: June 10/19

Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) calculate the "present value" of an annuity earning compound interest.

Quiz tomorrow

Last day's work: pp. 511-512 #2, 5a, 6, 7

p. 511 5. Calculate the future value of each annuity.

K

| | Regular Payment | Rate of Compound Interest per Year | Compounding Period | Time |
|----|----------------------|------------------------------------|--------------------|----------|
| a) | \$1500 per year | 6.3% | annually | 10 years |
| b) | \$250 every 6 months | 3.6% | semi-annually | 3 years |
| c) | \$2400 per quarter | 4.8% | quarterly | 7 years |
| d) | \$25 per month | 8% | monthly | 35 years |

5a)

$$S_{10} = \frac{1500((1.063)^{10} - 1)}{1.063 - 1}$$

$$= \frac{1500((1.063)^{10} - 1)}{0.063}$$

$$= 20\,051.963$$

$$= \$20\,051.96$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$a = 1500$$

$$r = 1 + i$$

$$i = \frac{0.063}{1}$$

$$= 0.063$$

$$\therefore r = 1.063$$

$$n = 10 \times 1$$

$$= 10$$

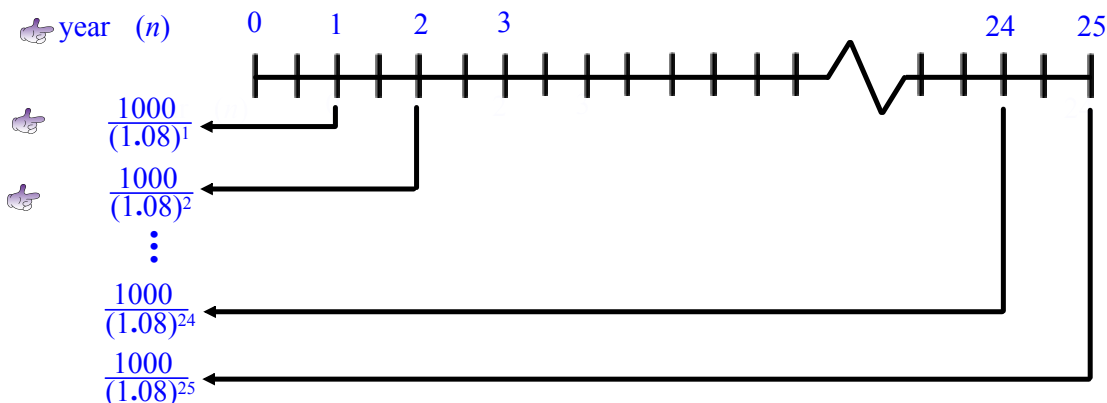
8.5 Annuities: Present Value

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Ex.1 In June, Ms. Sterlo decides to deposit some money into a fund to provide a \$1000 commencement scholarship at the end of each school year for the next 25 years.

The fund will pay 8% /a compounded annually. How much needs to be invested now?

$$A = P(1 + i)^n \quad \therefore P = \frac{A}{(1 + i)^n}$$



(It is easier to start at the bottom; it makes "r" nicer.)

$$S_{25} = \frac{1000}{(1.08)^{25}} + \frac{1000}{(1.08)^{24}} + \dots + \frac{1000}{(1.08)^2} + \frac{1000}{(1.08)}$$

This is a **Geometric Series**, with $a = \frac{1000}{(1.08)^{25}}$, $r = 1.08$, $n = 25$

Use $S_n = \frac{a(r^n - 1)}{r - 1}$

$$\therefore S_{25} = \frac{\frac{1000}{(1.08)^{25}} (1.08^{25} - 1)}{1.08 - 1}$$

$$\approx 10\,674.776$$

$$= \$10\,674.78$$

\therefore a sum of \$10 674.78 invested now would provide a payment of \$1000 for each of the next 25 years.

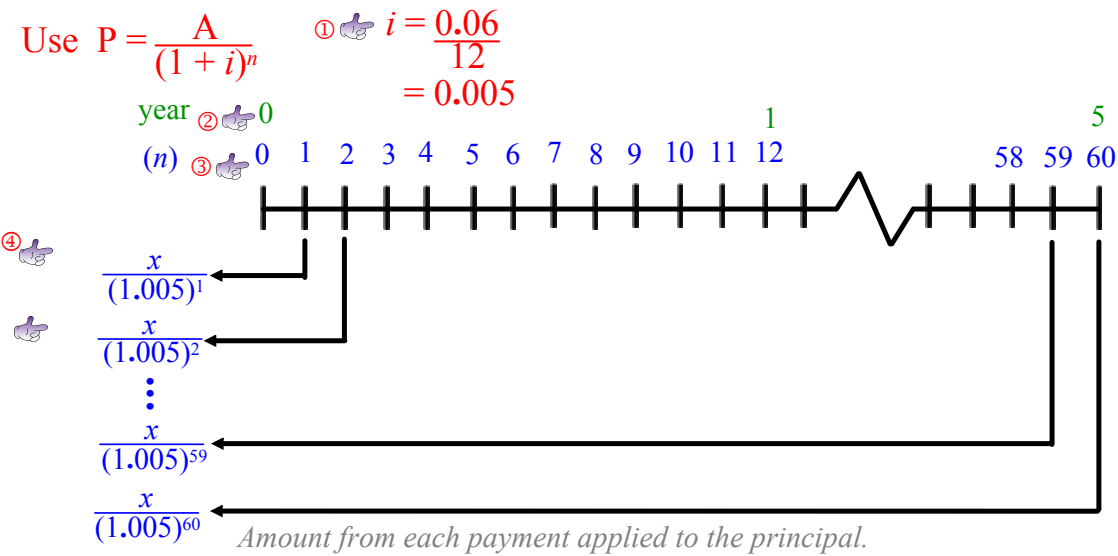
Discuss Interest earned?

$$\begin{array}{r} \$1000 \times 25 \\ = \$25\,000 \\ - \$10\,674.78 \\ \hline \$14\,325.22 \end{array}$$

Ex.2

At the end of university you owe \$20 000.

Determine your monthly payment at 6% /a compounded monthly for 5 years.



⑤ Use $S_n = \frac{a(r^n - 1)}{r - 1}$ ⑥ $a = \frac{x}{(1.005)^{60}}$, $r = 1.005$, $n = 60$, $S_{60} = 20\,000$

⑦ $20\,000 = \frac{\frac{x}{(1.005)^{60}} (1.005^{60} - 1)}{1.005 - 1}$ ← .005

Solve for x .

$20\,000 (.005) = \frac{x}{(1.005)^{60}} (1.005^{60} - 1)$

$20\,000 (.005) (1.005)^{60} = x (1.005^{60} - 1)$

$\frac{20\,000 (.005) (1.005)^{60}}{(1.005^{60} - 1)} = x$

$x \doteq 386.656$

$= \$386.66$

\therefore the monthly payment is \$386.66

If time, discuss Interest paid, and my "offer"?

$\$386.66 \times 60$
 $= \$23\,199.60$

Another formula:

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$

where R is the regular payment

i is the interest rate per compound period

n is the number of compound periods

Read p. 518 Example 3

Read the Key Ideas/Need to Know p.519

Today's Homework Practice includes:

pp. 520-521 #1, 2b, 3ac, 5, 7

Quiz tomorrow