Before we begin, are there any questions from last day's work?
Worksheet 1.2.4

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) solve problems involving exponential equations graphically, including problems arising from real-world context.
b) find the point of intersection of two exponential functions.
c) explain the solution in terms of real-life context.

1.3.2: Comparing Growths

Date: _Sept. 9/19_

Investigation

Audrey invested $1000 at 9% per annum compounded annually. Her daughter Brooke invested $2000 at 2.5% per annum compounded annually at the same time. How long did it take for the investments to be of equal value?

Materials:

desmos software, Formula for Amount of an Investment: $A = P(1+i)^n$

Method: *Number Questions*

1.   The equation for the amount of Audrey's investment is ☞ $A=1000(1+0.09)^n$

2.   The equation for the amount of Brooke's investment is ☞ $B=2000(1+0.025)^n$

3.   What type of function is each of the above? ☞ each is an exponential function

4.   Describe the expected shape of the graph of each.
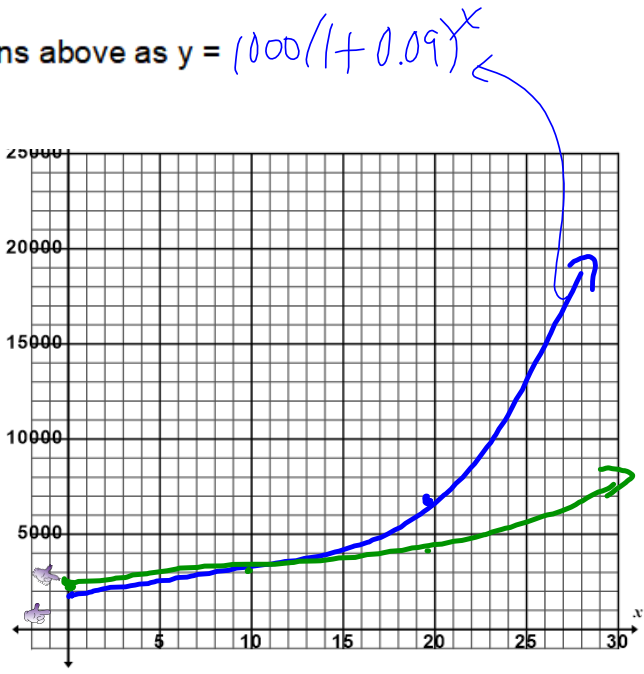   ☞ Each graph will be an ***increasing*** exponential curve.

5. Using desmos, enter the two equations above as y = $1000(1+0.09)^x$ ←

   and use the window settings below:

   ☑ X-Axis   [add]

   $-2 \leq x \leq 30$

   ☑ Y-Axis   [add]

   $-1000 \leq y \leq 25000$

   Sketch the graph from desmos on the grid at the right.
   (Possible alternate grid)
   (Uncover below)

   To find the point of intersection, click on one of the curves.

6. The point of intersection is ☞ $(11.27, 2641.95)$.

7. Explain the significance of the point of intersection ***in relation to the question***
    ☞ It took 11.27 years for both investments to be equal in value.

                                        (See next screen before hwk)

$(11.273, 2641.95)$
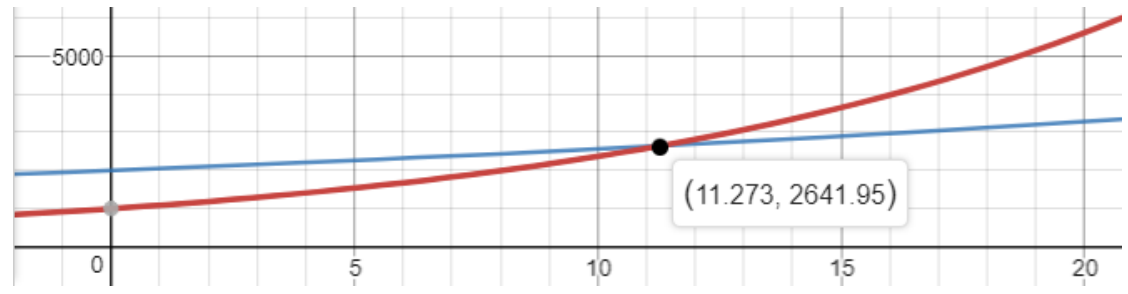
Double check student's understanding:

What if you invested $3000 at 12%/$a$ compounded semi-annually for 4 years?
Determine the values of A, P, i, and n.

A = ?

P = 3000

i = $\dfrac{0.12}{2}$ ✗

n = 4×2 ✗

$$A = 3000\left(1 + \frac{0.12}{2}\right)^8$$

Review the learning goals. Were we successful today?
Homework: Worksheet 1.3.3

Answer any remaining homework questions
Students ask for "at desk" clarification.

1.3.3: Crossing Curves   Date: _____

Use desmos to answer the following questions.
Adjust the window settings as appropriate for each question.

1. Determine the point of intersection of each pair of functions graphically.

a) $y = 2^{x+4}$ and $y = 2^7$
Settings   (-6 ≤ x ≤ 6, -50 ≤ y ≤ 500)

d) $f(x) = 6^{-x}$ and $f(x) = 8^{x+3}$
Settings   (-6 ≤ x ≤ 6, -5 ≤ y ≤ 50)

b) $y = 9^6$ and $y = 27^x$
Settings   (-6 ≤ x ≤ 6, -50 000 ≤ y ≤ 700 000)

e) $y = 3^{x+15}$ and $y = 27^{2x}$
Settings   (-6 ≤ x ≤ 6, -70 000 000 ≤ y ≤ 600 000 000)

c) $f(x) = 6^{-x}$ and $y = 36^5$
Settings   (-12 ≤ x ≤ 6, --50 000 ≤ y ≤ 70 000 000)
OR   (-12 ≤ x ≤ 6, --5x10^6 ≤ y ≤ 7x10^7)

f) $y = -x + 1$ and $y = 6^{-x}$
Settings   (-6 ≤ x ≤ 6, -0.5 ≤ y ≤ 1.5)

2a) Consider question 1(a) and the solution you determined.
How is the solution related to the expressions given for the exponents?

$$y = 2^{x+4} \qquad y = 2^7 \qquad \Rightarrow \therefore x + 4 = 7$$
$$2^{x+4} = 2^7 \qquad\qquad x = 3$$

b) Suggest a rule for solving exponential equations without graphing.

☞ **Use the exponents laws, get equal bases, then equate the exponents.**

ex) $9^6 = 27^x$

c) Can you solve questions (b) through (e) in the same way? Why or why not?

$$b, c, \& e \rightarrow Yes \qquad d \& f \rightarrow No$$

3. Al has saved $5000. He checked the website of a prominent bank. The rate for a savings account is 0.05% per annum, while the rate for a GIC is 3.85% per annum, both compounded annually. Al doesn't believe he wants to invest all $5000 for 5 years. He compared saving $5000 in the savings account to saving $4500 in the GIC. How long will it take for the investments to be equal in value?

☞ $A = 5000(1 + 0.0005)^n$

☞ $G = 4500(1 + 0.0385)^n$

4. The SarJen marketing company has determined that the effect on customers of a particular advertising campaign is modelled according to the function $A = 100(1.7^{-0.08x})$, where x is the time in weeks since the end of the advertising campaign and A is the value on their advertising rating scale.
Calculate the number of weeks until the effect of the advertising will fall to half (or a rating of 50)
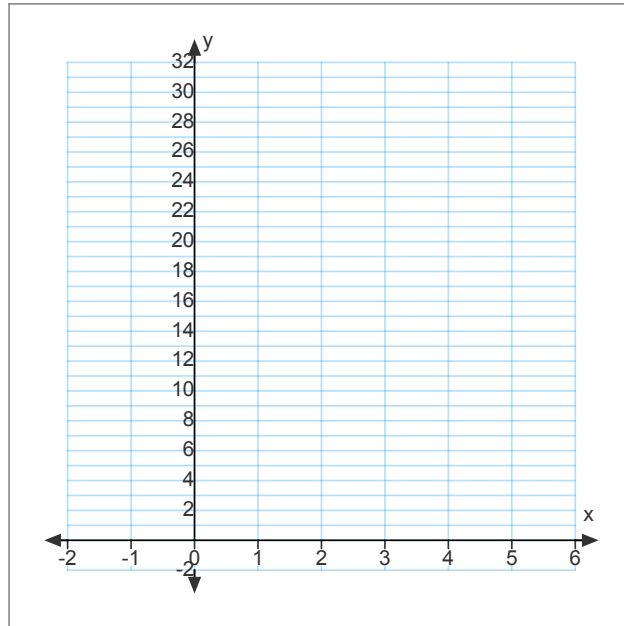[represent the 50 with y = 50 as function #1 using desmos.

$$\Rightarrow y = 100(1.7^{-0.08x})$$
$$y = 50$$
$$\therefore 16.328$$
$$\therefore \text{after 16 weeks.}$$

5. For the following system of equations find the point of intersection.

i) $y = 2x$

ii) $y = x^2$

iii) $y = 2^x$

Check that the point of intersection found is actually a point on all three functions.
Describe the rate of increase for each of the three functions.

Answers
1a) (3, 128) b) (4, 531 441) c) (-10, 60 466 000) or (-10, 60 466 176)   d) (-1.611, 17.946)
e) (3, 387 420 000)   f) (0, 1) **AND** (0.729, 0.271)
2) [a,b,c,e: Yes], [d,f: Not Possible]   **3**) 2.826 years   **4)** 16.328 weeks   **5**) (2, 4)