Before we begin, are there any questions from last day's work?

pp.352-353 #1(a,c),2(i,iii),3(a,b,c),4(a,b),5(a,b,c),8[**blue**](a,b,d),9(b,c)

"Show What You Know: 1.3" is first...

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) review all ideas for the unit summative.

Today's work

5.6

pp. 393-394 #1, 3-6, 7b, 8, 10(a,b)
Challenge Problem #15

Please submit the homework sheet 1.8.2 and p.344 #9, using the link in our Google Classroom.

1. Suppose a principal of P dollars is invested at 3.75% compounded annually. After n years, the amount is \$5000. This situation is modelled by the equation $P = 5000(1.0375)^{-n}$, where P is the present value.
   a) How much should be invested today to have \$5000 after 10 years?
   b) Suppose \$3000 are invested today. How long will it take until the amount is \$5000?

$P = 5000(1.0375)^{-n}$

$P = \dfrac{5000}{(1.0375)^n}$

$P(1.0375)^n = 5000$

$5000 = P(1.0375)^n$

$A = P(1+i)^n$

a) if $n = 10$

$P = \dfrac{5000}{(1.0375)^{10}}$

$\doteq 3460.102$

$\doteq \$3460.10$

b) if $P = 3000$

$3000 = 5000(1.0375)^{-n}$

$\dfrac{3000}{5000} = 1.0375^{-n}$

$\log 0.6 = -n \log 1.0375$

$\dfrac{-1 \log 0.6}{\log 1.0375} = n$

$n \doteq 13.87$

$\therefore 13.9$ years.

5. Evaluate each logarithm.
   a) $\log 1$
   b) $\log 10\,000$
   c) $\log_3 729$
   d) $\log_9\left(\frac{1}{9}\right)$
   e) $\log_4 0.0625$
   f) $\log_2 0.125$

a) $\log_{10} 1 = y$

$\therefore 10^y = 1$

$y = 0$

$\therefore \log 1 = 0$

b) $\log 10000 = 4$

$\log_{10} 10000 = y$

$10^y = 10000$

$y = 4$

c) $\log_3 729 = 6$

if $\log_3 729 = y$

$3^y = 729$

d) $\log_9\left(\frac{1}{9}\right) = y$

$9^y = \frac{1}{9}$ or $y = \dfrac{\log\left(\frac{1}{9}\right)}{\log 9}$

$9^y = 9^{-1}$ $\quad = -1$

$\therefore y = -1$

$\log_B A = \dfrac{\log A}{\log B}$

$\log_B A^n = n \log_B A$

6. Simplify each expression.

a) $\log 10^4$

$= \log 10000$

$= 4$

b) $\log_4 4^5$

$= 5\log_4 4$

$= 5(1)$

$= 5$

$\log_B B = 1$

c) $10^{\log 1000}$

$\log 1000$

$= 3$

$\therefore 10^{\log 1000}$

$= 10^3$

$= 1000$

d) $2^{\log_2 4}$

$= 2^2$

$= 4$

$\log_2 4 = 2$

$B^{\log_B A} = A$

8. The number of mutual funds available in Canada, M, is modelled by the equation $M = 460(1.19)^n$, where n is the number of years since 1989.

a) When will the number of mutual funds reach 10 000?

b) How many years will it take for the number of mutual funds to triple?

a) $10\,000 = 460(1.19)^n$

$\frac{10000}{460} = 1.19^n$

$\log\left(\frac{10000}{460}\right) = n \log 1.19$

$n = \frac{\log\left(\frac{10000}{460}\right)}{\log 1.19}$

$\doteq 17.7$

$\therefore 1989 + 17.7$

$\doteq 2006.7$

b) $460 \times 3 = 1380$

$m = 460(1.19)^n$

$1380 = 460(1.19)^n$

$\frac{1380}{460} = 1.19^n$

$3 = 1.19^n$

$\log 3 = n \log 1.19$

$n = \frac{\log 3}{\log 1.19}$

$\doteq 6.31$

$\therefore$ it will take 6.3 years to triple.

10. Radioactive tritium has a half-life of 12 years. A sample of this material has a mass of 1000 g. An equation that models the mass, m grams, remaining after t years is $m = 1000(0.9439)^t$.

a) How much radioactive tritium remains after 100 years?

b) How long does it take until only 100 g of the radioactive tritium remain?

a) $m = 1000(0.9439)^{100}$

$\doteq 3.1\text{ g}$

b) $100 = 1000(0.9439)^t$

$\frac{100}{1000} = 0.9439^t$

$\log(0.1) = t\log 0.9439$

$t = \frac{\log 0.1}{\log 0.9439}$

$\doteq 39.88$ years.

b₂) $P = P_0(0.5)^n$

$100 = 1000(0.5)^{\frac{t}{12}}$

$0.1 = 0.5^{\frac{t}{12}}$

$\log 0.1 = \frac{t}{12}\log 0.5$

$\frac{\log 0.1}{\log 0.5} = \frac{t}{12}$

$t = \frac{12\log 0.1}{\log 0.5}$

$\doteq 39.86$

15. Two historical purchases of land in North America are given. In each case, if the money had been invested at 6% compounded annually, what would its value be today?

a) In 1867, the United States purchased Alaska from Russia for \$7 200 000.

b) In 1626, Manhattan Island was sold for \$24.

$A = 24\left(1 + \frac{0.06}{1}\right)^{1x}$

$= 24(1.06)^{2019-1626}$

$= 24(1.06)^{393}$

$= 2.11 \times 10^{11}$

2 110 000 000 00

211 000 000 000

a) $x = 152$

$A = 7\,200\,000(1.06)^{152}$

$\doteq 5.05 \times 10^{10}$

A 1. Suppose you invest $200 at 6% compounded annually. How many years would it take for your investment to grow to each amount?

a) $300 b) $400 c) $600

$A = P(1+i)^n$

$A = 600$ $600 = 200(1+0.06)^x$

$P = 200$ $\frac{600}{200} = 1.06^x$

$i = \frac{0.06}{1}$ $3 = 1.06^x$

$n = 1x$ $\log 3 = x \log 1.06$

$\frac{\log 3}{\log 1.06} = x$

$x \doteq 18.85$

5. Calculate the number of years for an investment of $1000 to double at an interest rate of 7.2% for each compounding period.

a) annually **b)** semi-annually **c)** monthly **d)** daily

$A = 2000$

$P = 1000$

$i = \frac{0.072}{2}$

$n = 2x$

$A = P(1+i)^n$

$2000 = 1000\left(1+\frac{0.072}{2}\right)^{2x}$

$2 = \left(1+\frac{0.072}{2}\right)^{2x}$

$\frac{\log 2}{\log(1.036)} = 2x$

$\frac{\log 2}{2\log(1.036)} = x$

$x \doteq 9.79$ → $x \doteq 10$ years.

8. For every metre below the surface of water, the intensity of three colours of light is reduced as shown.

a) For each colour, write an equation to express the percent, P, of surface light as a function of the depth, d metres.

b) For each colour, determine the depth at which about one-half the light has disappeared.

c) Write each equation in part a as an exponential function with base 2.

d) For all practical purposes, the light has disappeared when the intensity is only 1% of that at the surface. At what depth would this occur for each colour?

Colour	Percent reduction (per metre)
Red	35%
Green	5%
Blue	2.5%

9. Polonium-210 is a radioactive element with a half-life of 20 weeks. From a sample of 25 g, how much would remain after each time?

a) 30 weeks   b) 14 weeks   c) 1 year   d) 511 days

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Today's work

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