Before we begin, are there any questions from last day's work?

6.7

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use a cubic model to solve a problem *with* technology.

p.224

4. A company that charters a boat for tours around the Gulf Islands can sell 200 tickets at $50 each. For every $10 increase in the ticket price, 5 fewer tickets will be sold.
   a) Represent the number of tickets sold as a function of the selling price.
   b) Represent the revenue as a function of the selling price.
   c) Sketch the function. What selling price will provide the maximum revenue? What is the maximum revenue?

p.224

6. Computer programs are sold to students for $25 each. Two hundred students are willing to buy them at that price. For every $5 increase in price, there are 20 fewer students willing to buy the software.
   a) Represent the sales revenue as a function of the price. Sketch the function.
   b) What is the maximum revenue?
   c) What range of prices will give a sales revenue that exceeds $5400?

a) Revenue = Number Sold × Price

$= (300-4p)p$

$= -4p^2+300p$

$= -4(p^2-75p)$

$= -4\left(p^2-75p+\left(\frac{75}{2}\right)^2-\left(\frac{75}{2}\right)^2\right)$

$= -4\left(p-37.5\right)^2-4\left(\frac{-5625}{4}\right)$

$= -4(p-37.5)^2+5625$

Number sold $= 200-\frac{4}{20}\left(\frac{p-25}{5}\right)$

$= 200-4(p-25)$

$= 200-4p+100$

$= 300-4p$

Axis: $x=\frac{-b}{2a}$

$=\frac{-(300)}{2(-4)}$

$=\frac{-300}{-8}$

$= 37.5$

p.224

7. The daily profit, P dollars, of a cotton candy vendor at the fair is described by the function $P = -60x^2 + 240x - 80$, where x dollars is the selling price of a bag of cotton candy.
   a) What should the selling price of a bag of cotton candy be to maximize daily profits?
   b) What is the maximum daily profit?

$x=\frac{-b}{2a}$

$=\frac{-240}{2(-60)}$

$= 2$

$P = -60(2)^2+240(2)-80$

$= -240+480-80$

$= \$160$

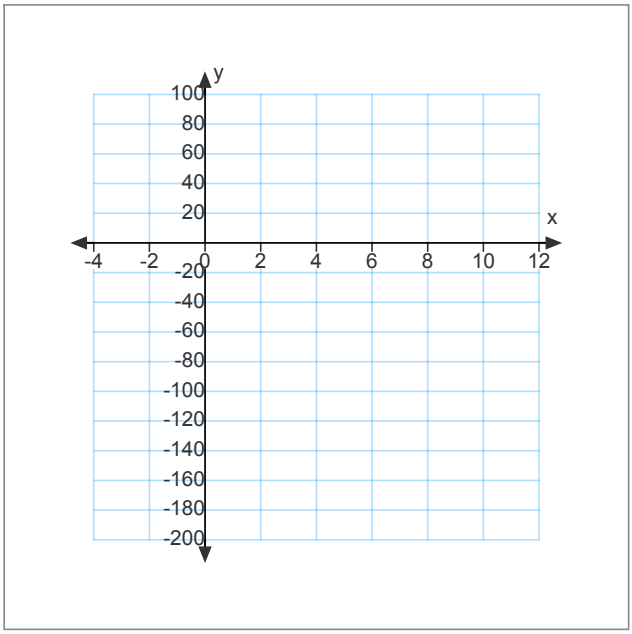
Introduction

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

$$y = -5.25\ (t - 4)^2 + 86$$

from - 4 to 12

$$y = -5.25(x - 4)^2 + 86$$



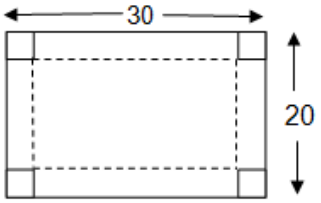
Modelling Using Cubic Functions

2.9.1: If You Build It...

Date: _____

1. Each member of your group should
   - cut out a 20 by 30 rectangle using grid paper,
   - cut 4 equal squares from each corner of the rectangle
     (one person could cut a 1 × 1 piece out of each corner; another use 2 × 2, etc.)

   NOTE: Each member of your group should have a different size of square that is being cut from the four corners,
   - fold the paper to create an open box (fold along dotted lines),
   - determine the dimensions of their box and record the information in the table.



2. Gather the data from the group and record your results in the table below.

Side Length of Square	Length of Box	Width of Box	Height of Box	Volume of Box
1	28	18	1	504
2	26	16	2	832
3	24	14	3	1008
4	22	12	4	1056
x	$30-2x$	$20-2x$	x	

3. Express the volume as a function of its side length

   $V(x) =$ $x(30-2x)(20-2x)$

4. Enter the function as Y_1 using desmos.

5. Which of the following would be the best choice for Xmin and Xmax? Justify your choices.

Xmin	Xmax	Justification
☐ -200	☐ -200	
☒ -10	☐ -10	
☐ 20	☒ 20	

6. Which of the following would be the best choice for Ymin and Ymax? Justify your choices.

Ymin	Ymax	Justification
~~☐ -500~~ -400	☐ -500	
☐ 10	☐ 10	
☐ 1500	~~☐ 1500~~ 1200	

7. Fill in the values for Xmin, Xmax, Ymin and Ymax into the window at right.

☑ X-Axis add a label

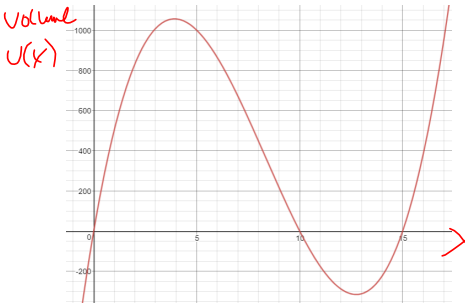
$-5 \le x \le 20$ Step:

☑ Y-Axis add a label

$-100 \le y \le 1200$ Step:


X-Axis
add a
$-5 \leq x \leq 20$
Y-Axis
add a
$-400 \leq y \leq 1200$
X-Axis
add a
$\leq x \leq$
Y-Axis
add a
$\leq y \leq$

8. The graph produced should appear as below.
Label your axes using words and variables.



9. What limitations are there on the size of the square that you can cut from each corner?

We have to cut Less than 10 cm.

10. State the domain and range of the volume function $V(x) =$ $x(30-2x)(20-2x)$

Domain: $\{x \in R \mid 0 < x < 10\}$ Range: $\{y \in R \mid 0 \leq y \leq 1056.306\}$

How are the restrictions evident from the	Domain	Range
box you are constructing?	Can't cut more than 10 cm from the width	
equation of the function?	$20-2x$ if $x=10$ $V(x)=0$	
graph of the function?	x-intercepts are 0 and 10	max value is 1056.306

11. From which model in #10 do you find easiest to identify the domain and range? Support your choice.

I think the graph is easiest, due to its visual nature.

12. The volume function from #10 is expressed in **function notation**.
In this scenario, V(2.5) means to determine the volume of the box when $x = 2.5$.
This is done by substituting 2.5 for every occurrence of x.
Use your function in question #10 to evaluate the function when $x = 2.5$. Show all of your work.

$V(2.5) = (2.5)(30-2(2.5))(20-2(2.5))$

$= 2.5(25)(15)$

$= 937.5 \text{ cm}^3$

13. Graph the line $x = 2.5$

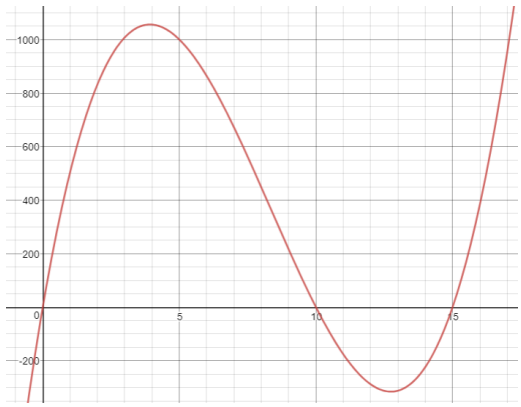
a) What is the value of the function when $x = 2.5$? 937.5 cm³

b) What do both of these values represent in the context of the box?

If the cut length is 2.5 cm, then the volume of the box is 937.5 cm².

14. Graph the function determine the value of the function when $x = 2.5$.

How is this represented on the graph of the function?



15a. Complete the following table.

Side Length	Point on Graph	Value of function V(x) =
$x = 0.5$	(0.5, 275.5)	V(0.5) =
$x = 4.5$	(4.5, 1039.5)	
$x = 2.7$	(2.7, 969.732)	

15b. Compare the points on the graph with the values of the volume found using substitution.

16. What are the dimensions that maximize the volume?

Maximum Volume	1056.3
Height of box at maximum	3.92
Length of box at maximum	22.15
Width of box at maximum	12.15

17. Using the graph, how do you know this is the maximum value?

18. How could you use the Volume function for $V(x)$ to show that the maximum occurs at the point you found?

19. If the initial dimensions of the box were doubled (i.e. 40 by 60), what do you think the maximum volume would be and the height of the box that gives that maximum?
Support your work by using a function and a graph.

A 1. A packaging company makes boxes using cardboard 25.0 cm long and 20.0 cm wide. Determine the size of squares to be cut from the corners for each of these boxes. Determine the dimensions of each box.

a) a box with volume 500 cm^3

b) a box with the maximum possible volume

2. Repeat exercise 1 for pieces of cardboard that are 25.0 cm square.

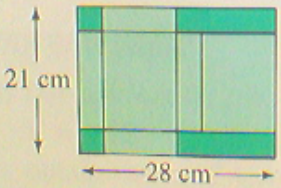
B 3. A packaging company is making another style of box from cardboard 28.0 cm long and 21.0 cm wide. This box has a top that comes from the same piece of cardboard. The diagram (below left) shows how it is made.

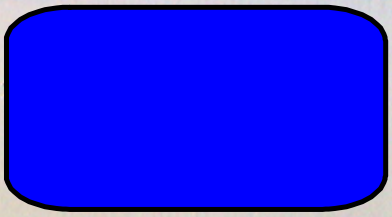
a) Let x centimetres represent the side length of each square cut from the corners. Write the volume of the box as a cubic function of x.

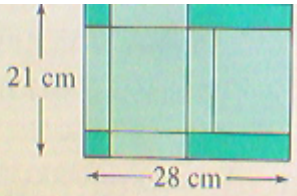
b) Graph the function in part a. Use a graphing calculator if you have one.

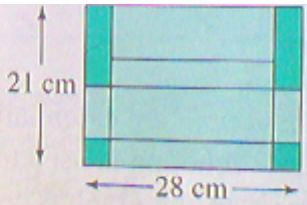
c) What size of square is cut from the corners to have a box with volume 375 cm^3? What are the dimensions of the box?

d) What size of square is cut from the corners to have a box with the maximum volume? What are the dimensions of this box?









4. The diagram (above right) shows another way to make a box with a top from the same piece of cardboard. Repeat exercise 3 for this box.