Today's Learning Goal(s):

By the end of the class, I will be able to:

a) Compare the factored form of a quadratic function with its standard form.
b) Find the equation of a quadratic curve given a vertex and the x-intercepts (roots).

MCF 3MI

3.2 Relating the Standard and Factored Forms (of Quadratic Functions)

Date: Oct. 8/19
(**Every** lesson)

Recall:

The vertex form of a Quadratic Function looks like: $f(x) = a(x-h)^2 + k$ $V(h,k)$

The standard form of a Quadratic Function looks like: $f(x) = ax^2 + bx + c$ $c = y\text{-int.}$

New: The factored form of a Quadratic Function looks like: $f(x) = a(x-r)(x-s)$

Ex.1 Sketch $y = 2(x-1)(x-5)$.
Determine the zeros, **(the equation of)** the axis of symmetry, and the coordinates of the vertex.

$A \times B = 0$
$A = 0$ or $B = 0$

$y = 2(x^{A}-1)(x^{B}-5)$

$0 = 2(x-1)(x-5)$

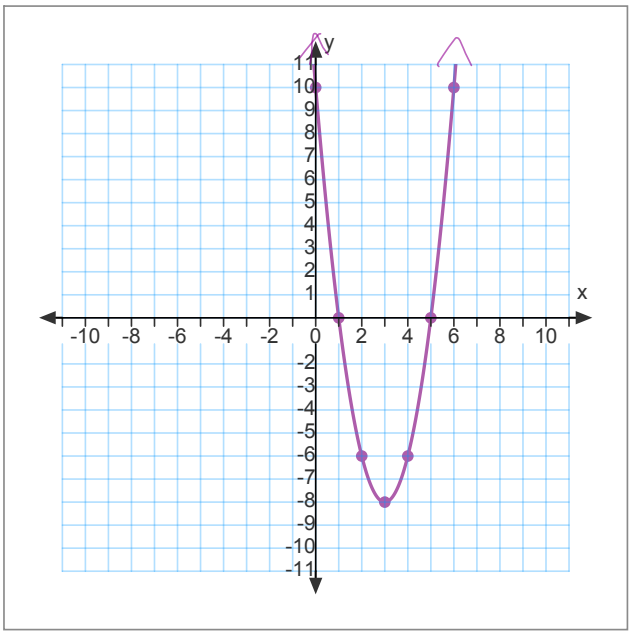
$x-1=0$ $\quad x-5=0$
$x=1$ $\quad x=5$

∴ the zeros are $x=1$ and $x=5$.

AoS: $x = \frac{1+5}{2}$

$= \frac{6}{2}$

$x = 3$



To find the vertex sub $x=3$ into the equation.

$y = 2(3-1)(3-5)$
$= 2(2)(-2)$
$= -8$ ∴ $V(3,-8)$

MG

$a = 2$

1 → 2
2 → 8
3 → 18

Ex. 2

For each quadratic function, determine the zeros, the equation of the axis of symmetry, and the coordinates of the vertex ***without*** graphing.

a) $f(x) = 2x(x-8)$

$0 = 2x(x-8)$

$2x = 0$ or $x - 8 = 0$

$x = 0$ $x = 8$

AOS: $x = \frac{0+8}{2}$

$x = 4$

$f(4) = 2(4)(4-8)$

$= 2(4)(-4)$

$= -32$

$\therefore V(4, -32)$

b) $g(x) = (x-9)(x+5)$

$0 = (x-9)(x+5)$

$x - 9 = 0$ or $x + 5 = 0$

$\therefore x = 9$ or $x = -5$

AOS: $x = \frac{9+(-5)}{2}$

$= \frac{4}{2}$

$x = 2$

$g(2) = (2-9)(2+5)$

$= (-7)(7)$

$= -49$

$\therefore V(2, -49)$

NOTE:

The axis of symmetry is equidistant from the two zeros!
Divide the distance between the zeros by 2 and
this is the value needed for the "axis of symmetry".

FYI: *x*-intercepts = zeros = roots = solutions

Ex. 3

Express each quadratic function in factored form. Then determine the zeros, the axis of symmetry, and the coordinates of the vertex ***without*** graphing.

a) $f(x) = 2x^2 - 12x$

$= 2x(x-6)$

$0 = 2x(x-6)$

$x=0$ or $x=6$

AofS: $x = \frac{0+6}{2}$

$x = 3$

$f(3) = 2(3)(3-6)$

$= 2(3)(-3)$

$= -18$

$\therefore V(3,-18)$

b) $g(x) = -x^2 + 49$

$= -1(x^2 - 49)$

$0 = -1(x-7)(x+7)$

$x=7$ or $x=-7$

AofS: $x = \frac{-7+7}{2}$

$= \frac{0}{2}$

$x=0$

$g(0) = -(0)^2 + 49$

$= 49$ $\therefore$ vertex $(0, 49)$

c) $y = x^2 + 5x - 6$

$0 = (x-1)(x+6)$

$x=1$ or $x=-6$

AofS: $x = \frac{1+(-6)}{2}$

$x = \frac{-5}{2}$

or $x = -2.5$

$y = \left(\frac{-5}{2} - 1\right)\left(\frac{-5}{2} + 6\right)$

$= \left(\frac{-5}{2} - \frac{2}{2}\right)\left(\frac{-5}{2} + \frac{12}{2}\right)$

$= \left(\frac{-7}{2}\right)\left(\frac{7}{2}\right)$

$= \frac{-49}{4}$

$\therefore V\left(\frac{-5}{2}, \frac{-49}{4}\right)$

$y = (-2.5-1)(-2.5+6)$

$= (-3.5)(3.5)$

$= (-12.25)$

$\therefore V(-2.5, -12.25)$

Same

d) $y = 3x^2 + 24x + 36$

$= 3(x^2 + 8x + 12)$

$= 3(x+6)(x+2)$

$0 = 3(x+6)(x+2)$

$\therefore x=-6$ or $x=-2$

AofS: $x = \frac{-6+(-2)}{2}$

$= \frac{-8}{2}$

$x=-4$

$y = 3(-4+6)(-4+2)$

$= 3(2)(-2)$

$= -12$

$\therefore V(-4, -12)$

Ex. 4 Express each quadratic function in standard form. Determine the y-intercept.

a) $f(x) = 4x(x-5)$

$= 4x^2 - 20x + 0$

$y\text{-int} = 0$

$y = 4(0)^2 - 20(0)$

$= 0 - 0$

$= 0$

b) $g(x) = (x-6)(x+2)$

$= x^2 + 2x - 6x - 12$

$= x^2 - 4x - 12$

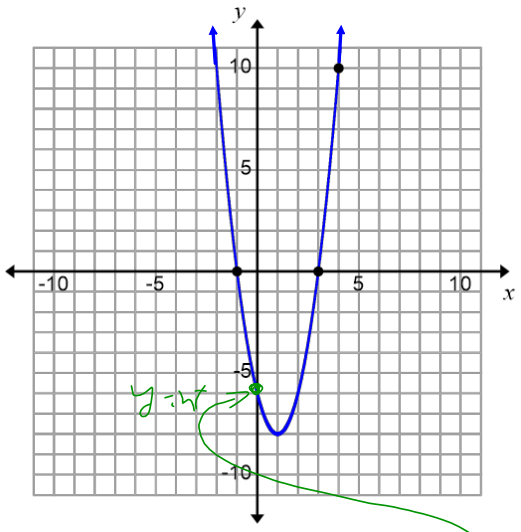
$y\text{-int} = -12$

$y = (0)^2 - 4(0) - 12$

$= 0 - 0 - 12$

$= -12$

Ex. 5 For the given graph, write the equation in both factored and standard forms.



$f(x) = a(x-r)(x-s)$

$x\text{-int}: x = -1$ and $x = 3$ pt $(4, 10)$ (x, y)

$f(x) = a(x+1)(x-3)$

$10 = a(4+1)(4-3)$

$10 = a(5)(1)$

$10 = 5a$

$\frac{10}{5} = a$

$\therefore a = 2$

$\therefore f(x) = 2(x+1)(x-3)$ is the eq'n in factored form

$f(x) = 2(x+1)(x-3)$

$= 2(x^2 - 3x + x - 3)$

$= 2(x^2 - 2x - 3)$

$f(x) = 2x^2 - 4x - 6$ is the equation in standard form.

Practice: pp. 139-142 # 2bd, 3cd, 4bde, 5be, 6, 7cde, 12bcd, 14

READ pp.153-154