Today's Learning Goal(s):

Date: Oct. 10/19

By the end of the class, I will be able to:

a) classify the nature of the roots of a quadratic equation using the discriminant.

b) use the discriminant in problem solving situations.

Links

Last day's Assigned Practice: Quadratic Functions Worksheet #1 6a, 7, 8

(Solutions posted on website.)

Two day's ago Assigned Practice: pp. 177-178 #1ac, 2ac, 4ace, 5, 6ac, 9, 10, 13

Clarify Inverse question using next screen!!

Clarifying (previous) Inverse questions

p. 161 #4c,d

$y=-2(x-1)^2+7$

$y=7-2(x-1)^2$

4. Given $f(x) = 7 - 2(x-1)^2$, $x \geq 1$, determine
a) $f(3)$ b) $f^{-1}(x)$ c) $f^{-1}(5)$ d) $f^{-1}(2a+7)$

$x = 7-2(y-1)^2 \quad ; y \geq 1$

$x-7 = -2(y-1)^2 \quad ; y \geq 1$

$\frac{x-7}{-2} = (y-1)^2 \quad ; y \geq 1$

$+\sqrt{\frac{x-7}{-2}} + 1 = y \quad ; y \geq 1$

$\therefore f^{-1}(x) = +\sqrt{\frac{x-7}{-2}} + 1 \quad ; y \geq 1$

or $f^{-1}(x) = 1 + \sqrt{\frac{x-7}{-2}}$

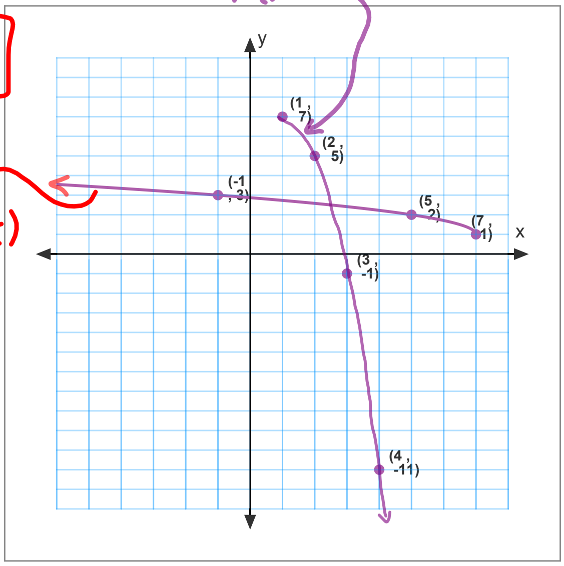
$= 1 + \sqrt{\frac{7-x}{2}}$ etc.

$f^{-1}(5) = 2$

$[\because f(2) = 5]$

f(x)

f⁻¹(x)



d) $f^{-1}(2a+7)$

$= 1 + \sqrt{\frac{(2a+7)-7}{-2}}$

$= 1 + \sqrt{\frac{2a}{-2}}$

$= 1 + \sqrt{-a} \quad [\therefore a \leq 0]$

$y = \sqrt{\frac{x-7}{-2}} + 1$

p. 161 #7

7. Given $f(x) = -(x+1)^2 - 3$ for $x \geq -1$, determine the equation for $f^{-1}(x)$. Graph the function and its inverse on the same axes.

$y=-(x+1)^2-3$

$f(x) = -(x+1)^2 - 3 \quad ; x \geq -1$

$x = -(y+1)^2 - 3 \quad ; y \geq -1$

$x+3 = -(y+1)^2$

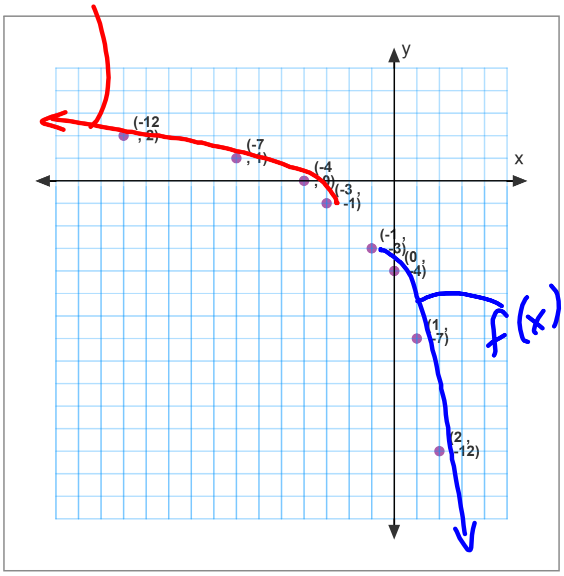
$-(x+3) = (y+1)^2$

$+\sqrt{-(x+3)} = y+1$

$\therefore y = +\sqrt{-(x+3)} - 1 \quad ; y \geq -1$

or $f^{-1}(x) = +\sqrt{-(x+3)} - 1$

f⁻¹(x)



f(x)

p. 177 **4. i)** For each equation, decide on a strategy to solve it and explain why you chose that strategy.

K

ii) Use your strategy to solve the equation. When appropriate, leave your answer in simplest radical form.

a) $2x^2 - 3x = x^2 + 7x$
b) $4x^2 + 6x + 1 = 0$
c) $x^2 + 4x - 3 = 0$
d) $(x + 3)^2 = -2x$
e) $3x^2 - 5x = 2x^2 + 4x + 10$
f) $2(x + 3)(x - 4) = 6x + 6$

e) $3x^2 - 5x = 2x^2 + 4x + 10$

$3x^2 - 2x^2 - 5x - 4x - 10 = 0$

$x^2 - 9x - 10 = 0$

$(x - 10)(x + 1) = 0$

$\therefore x = 10$ or $x = -1$

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

a) $P(x) = -x^2 + 12x + 28$
b) $P(x) = -2x^2 + 18x - 40$
c) $P(x) = -2x^2 + 22x - 17$
d) $P(x) = -0.5x^2 + 6x - 5$

c) When does $P(x)$ ← profit $= 0$ $\therefore$ break-even.

$0 = -2x^2 + 22x - 17$

$x = \dfrac{-22 \pm \sqrt{22^2 - 4(-2)(-17)}}{2(-2)}$

$= \dfrac{-22 \pm \sqrt{348}}{-4}$

$x \doteq 0.8631$ or $x \doteq 10.1636$

$\therefore x \doteq 863.1$ $\doteq 10163.6$

$\doteq 10164$

about 864 items sold or 10 164 items sold will break-even.

p. 178

9. A rectangle has an area of 330 m^2. One side is 7 m longer than the other. What are the dimensions of the rectangle?

$A = lw$

$330 = (w+7)w$

$0 = w^2 + 7w - 330$

$= (w+22)(w-15)$

$\therefore w = -22$ or $w = 15$

inadmissible

$l = w + 7$

$\therefore l = w + 7$
$= 15 + 7$
$= 22$

$\therefore$ the dimensions of the rectangle are 15 m × 22 m.

p. 178

10. The sum of the squares of two consecutive integers is 685. What could the integers be? List all possibilities.

Let x represent the first consecutive number.
Let $x+1$ represent the second consecutive number.

$x^2 + (x+1)^2 = 685$

$x^2 + x^2 + 2x + 1 = 685$

$2x^2 + 2x - 684 = 0$

$2(x^2 + x - 342) = 0$

$2(x+19)(x-18) = 0$

$\therefore x = -19$ or $x = 18$

if $x = -19$, then
$x+1$
$= -19+1$
$= -18$

if $x = 18$,
then $x+1 = 19$

$\therefore$ the consecutive numbers are -19 & -18
<u>OR</u> 18 & 19.

* Make sure you understand this;
it comes up a lot in various forms.

p. 178

13. A small flare is launched off the deck of a ship. The height of the flare above the water is given by the function $h(t) = -4.9t^2 + 92t + 9$, where $h(t)$ is measured in metres and t is time in seconds.

a) When will the flare's height be 150 m?

b) How long will the flare's height be above 150 m?

a) Let $h(t) = 150$

$$150 = -4.9t^2 + 92t + 9$$

$$0 = -4.9t^2 + 92t + 9 - 150$$

$$0 = -4.9t^2 + 92t - 141$$

$$t = \frac{-92 \pm \sqrt{92^2 - 4(-4.9)(-141)}}{2(-4.9)}$$

$$= \frac{-92 \pm \sqrt{5700.4}}{-9.8}$$

$$\therefore t = \frac{-92 + \sqrt{5700.4}}{-9.8} \quad \text{or} \quad t = \frac{-92 - \sqrt{5700.4}}{-9.8}$$

$$\doteq 1.683 \qquad\qquad \doteq 17.091$$

$$\doteq 1.68 \qquad\qquad \doteq 17.09$$

the flare's height above the water will be 150 m at 1.68 seconds (on the way up) AND again at 17.09 seconds (on the way down).

b) the flare will be above 150 m

from $1.68 \rightarrow 17.09$

$$\therefore 17.09 - 1.68$$

$$\doteq 15.41$$

the flare's will be 150 m above the water for 15.41 seconds

3.6 The Zeros of a Quadratic Function

Date: Oct. 10/19

Ex. 1: Find the zeros of the following using the quadratic formula.

a) $f(x) = 2x^2 - 5x - 3$

$0 = 2x^2 - 5x - 3$

$a = 2 \quad b = -5 \quad c = -3$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$= \frac{5 \pm \sqrt{25 + 24}}{4}$

$= \frac{5 \pm \sqrt{49}}{4}$

$= \frac{5 \pm 7}{4}$

$x = \frac{5+7}{4}$ or $x = \frac{5-7}{4}$

$= \frac{12}{4} \qquad = \frac{-2}{4}$

$= 3 \qquad = -\frac{1}{2}$

b) $g(x) = 2x^2 - 12x + 18$

$a = 2 \quad b = -12 \quad c = 18$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(18)}}{2(2)}$

$= \frac{12 \pm \sqrt{144 - 144}}{4}$

$= \frac{12 \pm \sqrt{0}}{4}$

$x = \frac{12 + 0}{4}$ or $x = \frac{12 - 0}{4}$

$x = 3$ or $x = 3$

c) $h(x) = 2x^2 - 4x + 5$

$a = 2 \quad b = -4 \quad c = 5$

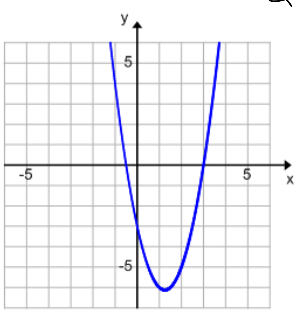
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(5)}}{2(2)}$

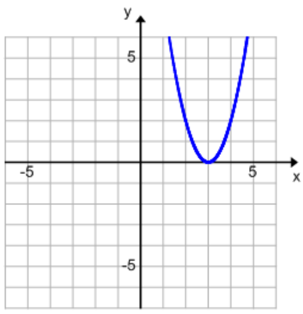
$= \frac{4 \pm \sqrt{16 - 40}}{4}$

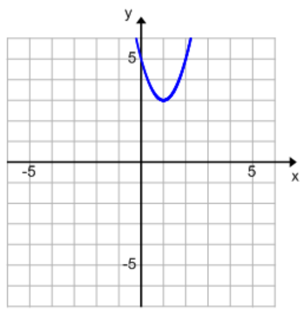
$= \frac{4 \pm \sqrt{-24}}{4}$

$\therefore$ No Real Solutions

click on the graphs







The Discriminant

For $ax^2 + bx + c = 0$ or for $f(x) = ax^2 + bx + c$
If $b^2 - 4ac > 0$, there are 2 solutions/zeros.
If $b^2 - 4ac = 0$, there is 1 solution/zeros.
If $b^2 - 4ac < 0$, there are no solutions/zeros.

Ex. 2: Determine the number of zeros for $f(x) = -3x^2 + 6x - 3$
What is the "nature of the roots" for $0 = -3x^2 + 6x - 3$?

$b^2 - 4ac$ $\quad a = -3 \quad b = 6 \quad c = -3$

$= (6)^2 - 4(-3)(-3)$

$= 36 - 36$

$= 0$

$\because b^2 - 4ac = 0$

$\therefore$ there is 1 solution

Ex. 3: For what values of k will the function $f(x) = 2x^2 + 4x + k$ have:

$a=2$ $b=4$ $c=k$

a) 1 zero?

$b^2 - 4ac = 0$

$(4)^2 - 4(2)(k) = 0$

$16 - 8k = 0$

$-8k = -16$

$k = \frac{-16}{-8}$

$= 2$

Inequality Rules?

b) 2 zeros?

$b^2 - 4ac > 0$

$(4)^2 - 4(2)(k) > 0$

$16 - 8k > 0$

$16 > 8k$

$\frac{16}{8} > k$

$2 > k$

$\therefore k < 2$

$-8k > -16$

$k < \frac{-16}{-8}$

$k < 2$

c) no zeros?

$b^2 - 4ac < 0$

$(4)^2 - 4(2)(k) < 0$

$16 - 8k < 0$

$-8k < -16$

$k > \frac{-16}{-8}$

$k > 2$

Ex. 4: For what value(s) of k will the function $g(x) = kx^2 + 8x + k$ have no real roots?

$a=k \quad b=8 \quad c=k$

$$b^2 - 4ac < 0$$
$$8^2 - 4(k)(k) < 0$$
$$64 - 4k^2 < 0$$
$$-4(k^2 - 16) < 0$$
$$-4(k+4)(k-4) < 0$$

Think: for LS <0, then both brackets must be positive. So, what value(s) of k will make the brackets positive.

For k+4>0 then k>–4
For k–4>0, then k>4

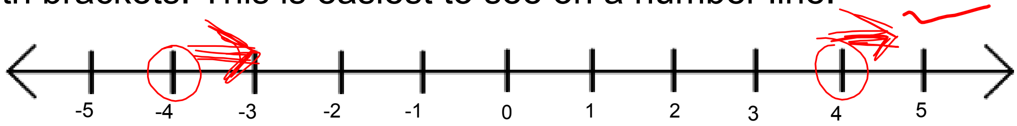
OR

For k+4<0 then k<–4
For k–4<0 then k<4

Since both brackets have to be positive or negative AT THE SAME TIME, then the value(s) of k have to "work" for both brackets. This is easiest to see on a number line.

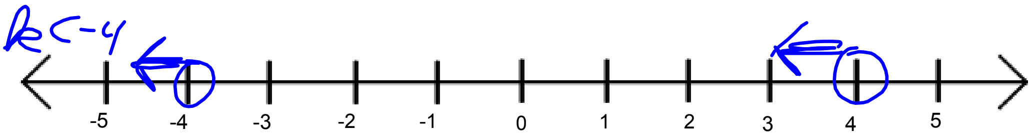
$\therefore k>4$

Both pos:

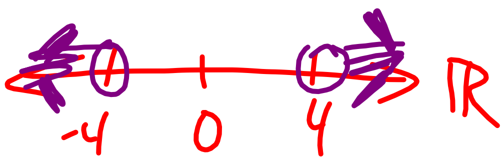


Both neg:

$k<-4$



if k>4 **or** k<–4, the function will have no real roots.



Are there any questions from last day's assigned work you would like to see on the board?

Last day's Assigned Practice: Quadratic Functions Worksheet #1

(Solutions posted on website.)

Two day's ago Assigned Practice: pp. 177-178 #1ac, 2ac, 4ace, 5, 6ac, 9, 10, 13

Fall 2019=Not Marked Yet

If time, return and correct SWYK 3.1

Today's Assigned Practice includes:

pp. 185-186 #1bde, 3ac, 4ac, 6, 7 [14,17,18]