

Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) review all ideas for the unit summative.

(See Next Page)

*You may wish to print out 4.1.1
in advance of next class.*

3.7.2 Connecting Formulae Worksheet 1

Extra practiceMake x the subject of the following:

- 1 $3x + 1 = y$ 2 $12 - 2x = 4y + 2$ 3 $y = \frac{x+1}{2}$
- 4 $3 + 4y + 2x = 9$ 5 $2xy + 1 = 3y$ 6 $2(x+3) = 3(x+2y-1)$
- 7 $\frac{x-1}{y+2} = 3$ 8 $y = \frac{2}{x+1}$ 9 $\frac{3-x}{y} = x$
- 10 $\frac{3}{x-1} = 5y$ 11 $\frac{xy}{x+1} = 3$ 12 $y-3 = \frac{x}{2x+1}$
- 13 $z = x^2y + 3$ 14 $xyz = \frac{3b}{x}$ 15 $(x+1)^2 = 3ab + 1$
- 16 $y = \sqrt{(2x+3)}$ 17 $\frac{x-z}{2} = \frac{x+y}{3}$ 18 $y = \frac{1}{\sqrt{x}}$
- 19 $z^2 = x^2 + y^2$ 20 $2(x+3) - 3(y+2) = 4xy$ 21 $z = \sqrt{xy} + 1$
- 22 $\frac{x}{y+1} = \frac{n}{x^2y}$ 23 $\frac{x+3y}{z-2x} = 3$ 24 $\frac{(x-1)^2}{4} + (2y+1)^2 = 1$

Answers

- 1 $x = \frac{y-1}{3}$ 2 $x = 5 - 2y$ 3 $x = 2y - 1$
- 4 $x = 3 - 2y$ 5 $x = \frac{3y-1}{2y}$ 6 $x = 9 - 6y$
- 7 $x = 3y + 7$ 8 $x = \frac{2}{y} - 1$ 9 $x = \frac{3}{y+1}$
- 10 $x = \frac{3}{5y} + 1$ 11 $x = \frac{3}{y-3}$ 12 $x = \frac{3-y}{2y-7}$
- 13 $x = \sqrt{\frac{z-3}{y}}$ 14 $x = \sqrt{\frac{3b}{yz}}$ 15 $x = \sqrt{(3ab+1)} - 1$
- 16 $x = \frac{y^2-3}{2}$ 17 $x = 2y + 3z$ 18 $x = \frac{1}{y^2}$
- 19 $x = \sqrt{z^2 - y^2}$ 20 $x = \frac{3y}{2-4y}$ 21 $x = \frac{(z-1)^2}{y}$
- 22 $x = \sqrt[3]{\frac{n(y+1)}{y}}$ 23 $x = \frac{3(z-y)}{7}$ 24 $x = 2\sqrt{1 - (2y+1)^2} + 1$

$$\textcircled{7} \quad \frac{x-1}{y+2} = 3$$

$$x-1 = 3(y+2)$$

$$x = 3(y+2)+1$$

$$x = 3y+7$$

$$\textcircled{10} \quad \frac{3}{x-1} = 5y$$

$$3 = 5y(x-1)$$

$$3 = 5xy - 5y$$

$$\frac{3+5y}{5y} = \frac{5xy}{5y}$$

$$\frac{3+5y}{5y} = x$$

$$\text{or } \frac{3}{5y} + 1 = x$$

$$\textcircled{12} \quad y-3 = \frac{x}{2x+1}$$

$$(2x+1)(y-3) = x$$

$$2xy - 6x + y - 3 = x$$

$$2xy - 6x - x = 3 - y$$

$$2xy - 7x = 3 - y$$

$$x(2y-7) = 3-y$$

$$x = \frac{3-y}{2y-7}$$

$$\textcircled{9} \quad \frac{3-x}{y} = x$$

$$3-x = xy$$

$$3 = x+xy \quad -x = y+3$$

$$3 = x(1+y) \quad -x - xy = 3$$

$$-x(1+y) = 3$$

$$-x = \frac{3}{1+y} \quad \text{or } x = \frac{3}{y+1}$$

$$\textcircled{11} \quad \frac{xy}{x+1} = 3$$

$$xy = 3(x+1)$$

$$xy = 3x+3$$

$$xy - 3x = 3$$

$$x(y-3) = 3$$

$$x = \frac{3}{y-3}$$

$$\textcircled{14} \quad xyz = \frac{3b}{x}$$

$$x(xyz) = x\left(\frac{3b}{x}\right)$$

$$x^2yz = 3b$$

$$x^2 = \frac{3b}{yz}$$

$$x = \pm \sqrt{\frac{3b}{yz}}$$

c) For $r = \frac{I}{Pt}$, find r when $I = 58.27$, $t = 5.4$, and $P = 500$.

$$\begin{aligned} r &= \frac{58.27}{500(5.4)} \\ &= 0.02158 \\ &= 2.158 \\ &= 2.16\% \end{aligned}$$

d) For $P = \frac{A}{(1+i)^n}$, find P when $A = 1500$, $i = 0.035$, and $n = 24$.

$$\begin{aligned} P &= \frac{1500}{(1+0.035)^{24}} \\ &= 656.935 \\ &= \$656.94 \end{aligned}$$

f) For $V = \frac{4}{3}\pi r^3$, find V when $r = 3.5$.

$$\begin{aligned} &= \frac{4}{3}\pi(3.5)^3 \\ &= 179.594 \\ &= 179.59 \end{aligned}$$

Some to try

- 1 Make I the subject of $V = IR$
- 2 Make m the subject of $y = mx + c$
- 3 Make h the subject of $V = \pi r^2 h$
- 4 Make r the subject of $V = \pi r^2 h$
- 5 Make u the subject of $v = u + at$
- 6 Make P the subject of $I = \frac{PRT}{100}$
- 7 Make W the subject of $P = 2(L + W)$
- 8 Make l the subject of $L = l(1 + at)$
- 9 Make h the subject of $A = \frac{h(a+b)}{2}$
- 10 Make a the subject of $A = \frac{h(a+b)}{2}$
- 11 Make m the subject of $I = mv - mu$
- 12 Make u the subject of $I = mv - mu$
- 13 Make h the subject of $A = 2\pi r^2 + 2\pi rh$
- 14 Make a the subject of $s = ut + \frac{1}{2}at^2$
- 15 Make d the subject of $F = \frac{L}{4\pi d^2}$
- 16 Make v the subject of $E = \frac{1}{2}cv^2$
- 17 Make T the subject of $c = \sqrt{\frac{T}{\mu}}$
- 18 Make C the subject of $F = \frac{2}{3}C + 32$
- 19 Make r the subject of $A = \pi(R^2 - r^2)$
- 20 Make l the subject of $T = 2\pi\sqrt{\frac{l}{g}}$
- 21 Make p the subject of $E^2 = p^2c^2 + m^2c^4$
- 22 Make R the subject of $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
- 23 Make v the subject of $E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$
- 24 Make v the subject of $p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$

Other answers are possible – if in doubt, ask your tutor.

- 1 $I = \frac{V}{R}$
- 2 $m = \frac{y-c}{x}$
- 3 $h = \frac{V}{\pi r^2}$
- 4 $r = \sqrt{\frac{V}{\pi h}}$
- 5 $u = v - at$
- 6 $P = \frac{100I}{RT}$
- 7 $W = \frac{P}{2} - L$ or $\frac{P-2L}{2}$
- 8 $l = \frac{L}{1+at}$
- 9 $h = \frac{2A}{a+b}$
- 10 $a = \frac{2A}{h} - b$ or $\frac{2A-hb}{h}$
- 11 $m = \frac{I}{v-u}$
- 12 $u = \frac{mv-I}{m}$
- 13 $h = \frac{A-2\pi r^2}{2\pi r}$
- 14 $a = \frac{2(c-bt)}{t^2}$
- 15 $d = \sqrt{\frac{L}{4\pi F}}$
- 16 $v = \sqrt{\frac{2E}{c}}$
- 17 $T = \mu c^2$
- 18 $C = \frac{3}{2}(F-32)$
- 19 $r = \sqrt{\frac{\pi R^2 - A}{\pi}}$
- 20 $l = \left(\frac{T}{2\pi}\right)^2 g$
- 21 $p = \sqrt{\frac{E^2 - m^2c^4}{c^2}}$
- 22 $R = \frac{R_1 R_2}{R_1 + R_2}$
- 23 $v = c \sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$
- 24 $v = \frac{1}{\sqrt{\frac{m^2}{p^2} + \frac{1}{c^2}}}$

3.7.3 Connecting Formulae Worksheet 2

Exercises

- Find the value of the indicated variable in each formula. Where necessary, give the answer to 2 decimal places.
 - For $p = 2(l + w)$, find p when $l = 13.9$ and $w = 7.2$.
 - For $A = \frac{1}{2}bh$, find A when $b = 5.9$ and $h = 3.4$.
 - For $I = Prt$, find I when $P = 525$, $r = 0.0375$, and $t = 5.5$.
 - For $P = 4s$, find P when $s = 64.75$.
 - For $V = \pi r^2 h$, find V when $r = 5.1$ and $h = 9.4$.
 - For $C = 2\pi r$, find C when $r = 17.5$.
 - For $A = 2\pi rh$, find A when $r = 6.2$ and $h = 7.3$.
- Find the value of the indicated variable in each formula. Where necessary, give the answer to 2 decimal places.
 - For $S = \frac{n(n+1)}{2}$, find S when $n = 87$.
 - For $A = \frac{h(a+b)}{2}$, find A when $a = 4.6$, $b = 5.7$, and $h = 6.4$.
 - For $r = \frac{I}{Pt}$, find r when $I = 58.27$, $t = 5.4$, and $P = 500$.
 - For $P = \frac{A}{(1+i)^n}$, find P when $A = 1500$, $i = 0.035$, and $n = 24$.
 - For $V = \frac{1}{3}\pi r^2 h$, find V when $r = 4.1$ and $h = 8.3$.
 - For $V = \frac{4}{3}\pi r^3$, find V when $r = 3.5$.

ANSWERS**Necessary Skills: 3 New—Evaluating a Formula Exercises**

- $p = 42.2$
 - $A = 10.03$
 - $I = 108.28$
 - $P = 259$
 - $V = 768.10$
 - $C = 109.96$
 - $A = 284.38$
- $S = 3828$
 - $A = 32.96$
 - $r = 0.02$
 - $r = 12.06$
 - $P = 656.94$
 - $V = 146.11$
 - $V = 179.59$

3. Given the formula $E = I^2 Rt$:

- Find R when $E = 1\,500$, $I = 4$ and $t = 50$, to one decimal place.
- Find t when $E = 2\,500$, $I = 3.5$ and $R = 30$, to one decimal place.
- Find I when $E = 1\,200$, $R = 10$ and $t = 35$, to one decimal place.

FINAL ANSWERS: a) 1.9 b) 6.8 c) 1.9

4. Solve for x :

- $A = Lx$
- $P = 2L + 2x$
- $y = mx + b$
- $A = \frac{1}{2}hx$

FINAL ANSWERS: a) $x = \frac{A}{L}$ b) $\frac{P - 2L}{2} = x$ c) $x = \frac{y - b}{m}$ d) $x = \frac{2A}{h}$

3.9.1 Review

Date: Oct. 21/19

1. Factor the following completely. Note: You are **NOT** solving for x.

a) $25x^8 - 30x^5 + 35x$

b) $144x^4 - 25z^2$

c) $7x(x+2) - 5(x+2)$

$= 5x(5x^7 - 6x^4 + 7)$

d) $x^2 - 10x - 24$

e) $x^2 - 12x + 32$

$= (x-12)(x+2)$

$x^{10} - 10x^5 - 24$
 $= (x^5 - 12)(x^5 + 2)$

$x^{10} + 2x^5 - 12x^5 - 24$
 $= x^{10} - 10x^5 - 24$

g) $7x^2 + x - 8$

h) $8x^2 - 5x - 3$

i) $x^4 - 3x^3 + 2x - 6$

D $3x^2 - x - 30$

$= 3x^2 + 9x - 10x - 30$
 $= 3x(x+3) - 10(x+3)$
 $= (3x-10)(x+3)$

Check Solns

S: -1	
P: -90	
1	90
2	45
3	30
4	
5	18
6	15
7	
8	
9 - 10	

j) $a^2 - 2a + ad - 2d$

k $2x^4 - 98x^2$

$= 2x^2(x^2 - 49)$
 $= 2x(x-7)(x+7)$

- 1a) $5x(5x^7 - 6x^4 + 7)$ b) $(12x^2 - 5z)(12x^2 + 5z)$ c) $(7x-5)(x+2)$ d) $(x-12)(x+2)$
 e) $(x-8)(x-4)$ f) $(3x-10)(x+3)$ g) $(7x+8)(x-1)$ h) $(8x+3)(x-1)$
 i) $(x^3+2)(x-3)$ j) $(a+d)(a-2)$ k) $2x^2(x-7)(x+7)$

2. Solve. Factor and use the quadratic formula where needed.

a) $y^3 + y^2 + 2y + 2 = 0$

$$y^2(y+1) + 2(y+1) = 0$$

$$(y+1)(y^2+2) = 0$$

$y+1=0 \Rightarrow y=-1$
 $y^2+2=0 \Rightarrow y^2=-2 \Rightarrow y = \pm\sqrt{-2}$ (No Real Solns)

b) $16x^2 - 36 = 0$

c) $15x^2 + 3x - 12 = 0$

$$3(5x^2 + x - 4) = 0$$

$$3[5x^2 - 4x + 5x - 4] = 0$$

$$3[x(5x-4) + 1(5x-4)] = 0$$

$$3(5x-4)(x+1) = 0$$

$x = \frac{4}{5} \quad \therefore x = -1$ Check Solns

d) $2x^4 - 18x^2 = 0$

e) $x^3 - 3x^2 + 2x = 0$

f) $2x^4 - 20x^3 + 48x^2 = 0$

g) $2x^2 + 13x + 15 = 0$

h) $x^3 - 19 = 0$

$$x^3 = 19$$

$$x = \sqrt[3]{19}$$

$$\approx 2.668$$

$$\approx 2.67$$

i) $-4x^3 + 10x^2 - 2x = 0$

$$-2x(2x^2 - 5x + 1) = 0$$

$x=0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25-8}}{4}$$

$$x = \frac{5 + \sqrt{17}}{4} \text{ or } x = \frac{5 - \sqrt{17}}{4}$$

$$\approx 2.28 \quad \approx 0.219$$

$$\approx 0.22$$

j) $x(x^2 - x - 2) = 14 - x(x+2)$

$$x^3 - x^2 - 2x = 14 - x^2 - 2x$$

$$x^3 - x^2 + x^2 - 2x + 2x - 14 = 0$$

$$x^3 - 14 = 0$$

$$x^3 = 14$$

$$x = \sqrt[3]{14}$$

$$\approx 2.410$$

$$\approx 2.41$$

k) $-4x^2 + 36 = -x^3 + 9x$

$$x^3 - 4x^2 - 9x + 36 = 0$$

$$x^2(x-4) - 9(x-4) = 0$$

$$(x-4)(x^2-9) = 0$$

$$(x-4)(x+3)(x-3) = 0$$

$\therefore x = 4, -3, 3$

2a) $y = -1$ b) $x = \frac{-3}{2}, \frac{3}{2}$ c) $x = \frac{4}{5}, -1$ d) $x = 0, -3, 3$ e) $x = 0, 1, 2$ f) $x = 0, 6, 4$

g) $x = \frac{-3}{2}, -5$ h) $x \approx 2.67$ i) $x = 0, x \approx 2.28, x \approx 0.22$ j) $x \approx 2.41$ k) $x = -3, 3, 4$

3. Find each product.

a) $(4x+5y)(8x-9y)$

b) $(-6x-3y)(-6x^2-xy+5y^2)$

Check Solns

$$\begin{aligned}
 & \text{c) } (x^2 - 3xy + 2y^2)(-2x^2 + 4xy + 5y^2) \\
 & = -2x^4 + 4x^3y + 5x^2y^2 + 6x^3y - 12x^2y^2 - 15xy^3 - 4x^2y^2 + 8xy^3 + 10y^4 \\
 & = -2x^4 + 10x^3y - 11x^2y^2 - 7xy^3 + 10y^4
 \end{aligned}$$

3a) $32x^2 + 4xy - 45y^2$ b) $36x^3 + 24x^2y - 27xy^2 - 15y^3$ c) $-2x^4 + 10x^3y - 11x^2y^2 - 7xy^3 + 10y^4$

4. Rearrange each formula.

a) Make x the subject of $5 + 8y + 4x = 33$

b) Make b the subject of $A = \frac{h(a+b)}{2}$

Check Solns

c) Make v the subject of $I = mv - mu$

d) Make x the subject of $\frac{x+3y}{z-2x} = 3$

$$x + 3y = 3(z - 2x)$$

$$x + 3y = 3z - 6x$$

$$x + 6x = 3z - 3y$$

$$7x = 3z - 3y$$

$$x = \frac{3z - 3y}{7}$$

e) Make x the subject of $(x+1)^2 = 4yz + 6$

$$x + 1 = \pm \sqrt{4yz + 6}$$

$$x = -1 \pm \sqrt{4yz + 6}$$

4a) $x = -2y + 7$

b) $b = \frac{2A}{h} - a$

c) $v = \frac{I}{m} + u$

d) $x = \frac{3z - 3y}{7}$

e) $x = \pm \sqrt{4yz + 6} - 1$

4. Melissa is running a ski trip during the exam break.
 The bus holds 40 students and if she charges \$250 per student the bus will be filled.
 For every \$25 increase in the price she charges students, two fewer students will go on the trip.
 a) Write an equation to model Melissa's revenue.
 b) Determine the maximum revenue.
 c) How many students need to go on the trip for Melissa to earn \$8800?

a) Revenue = price \times number of tickets
 $= p(-0.08p + 60)$
 $= -0.08p^2 + 60p$

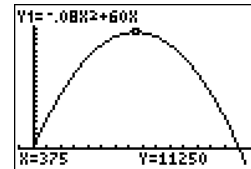
b) $R = -0.08p^2 + 60p$
 $= -0.08p(p - 750)$
 \checkmark or $p = 750$
 $p = 0$

$x = \frac{-b}{2a}$

$= 40 - 2 \left(\frac{p - 250}{25} \right)$
 $= 40 - \frac{2}{25}(p - 250)$
 $= 40 - \frac{2}{25}p + 20$
 $= -\frac{2}{25}p + 60$
 $= -0.08p + 60$

the max. revenue would occur when ticket price is set at \$375
 (the max. revenue would be \$11250, from 30 tickets being sold. $11250 \div 375 = 30$)

c) $8800 = -0.08p^2 + 60p$



$0.08p^2 - 60p + 8800 = 0$

$0.08(p^2 - 750p + 11000) = 0$

$0.08(p - 550)(p - 200) = 0$

$p = 550$ or $p = 200$

to earn a revenue of \$8800, ticket price must be set at \$550

(resulting in only 16 tickets being sold $8800 \div 550 = 16$)

or the ticket price must be set at \$200

(resulting in 44 tickets needing to be sold $8800 \div 200 = 44$)

[What is the problem with this idea?]

Melissa is best off setting the price at \$375

