Before we begin, are there any questions from last day's work? **3.5.1 and ~~#2~~**

Today's Learning Goal(s):

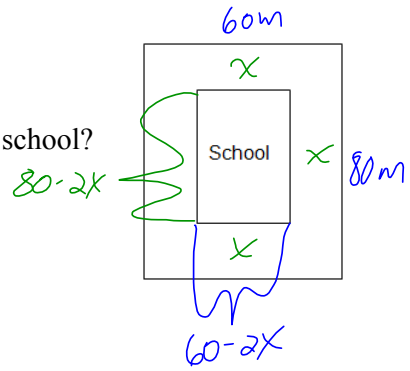
By the end of the class, I will be able to:

a) solve problems algebraically that involve polynomial functions & equations arising from real-world applications

3.6.1 Applications of Polynomial Equations

Date: Oct. 17/19

1. A school is to be built on a rectangular lot measuring 80 m by 60 m.
   A lawn of uniform width is to surround the school.
   The area of the lawn is equal to the area of the school.
   How wide will the strip of lawn be **AND** what are the dimensions of the school?



Solution

Let x represent the width of the **lawn**, in m.
Let $80-2x$ represent the length of the school, in m.
Let $60-2x$ represent the width of the school, in m.

$A = \ell w$

$A_{School} = (80-2x)(60-2x)$

$= 4800 - 160x - 120x + 4x^2$

$2400 = 4x^2 - 280x + 4800$

$0 = 4x^2 - 280x + 4800 - 2400$

$0 = 4x^2 - 280x + 2400$

$= 4(x^2 - 70x + 600)$

$= 4(x-60)(x-10)$

$\therefore x = 60$ or $x = 10$

inadmissible

↳ if $x = 60$, the dimensions of the lot are negative values.

$A_{School} = A_{lawn}$

$A_{lawn} = A_{total} \div 2$

$= \frac{(80)(60)}{2}$

$= 2400$

$= A_{School}$

$A_{total} = (80)(60)$

$(A_{lot}) = 4800\ m^2$

$\therefore$ the width of the strip will be 10 m.

$\ell = 80-2x$ $\quad w = 60-2x$

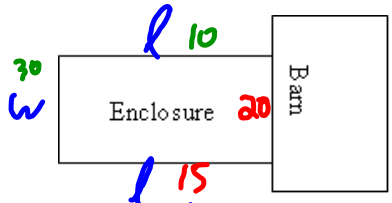
$= 80-2(10)$ $\quad = 60-2(10)$

$= 80-20$ $\quad = 60-20$

$= 60$ $\quad = 40$

$\therefore$ the dimensions of the school are 60 m x 40 m

2. A farmer needs to enclose a rectangular area using 50 m of fencing.
One of the sides of the enclosure is against the barn.
If the area of the enclosure is 300m², determine the dimensions of the enclosure.



Let l + w represent the length + width respectively in m.

$A = lw$ $\quad 2l + w = 50$

$300 = lw$ $\quad w = 50 - 2l$

$300 = l(50 - 2l)$

$300 = 50l - 2l^2$

$2l^2 - 50l + 300 = 0$

$2(l^2 - 25l + 150) = 0$

$2(l - 10)(l - 15) = 0$

$\therefore l = 10$ or $l = 15$

if $l = 10$ m

then $w = 50 - 2(10)$

$= 30$ m

if $l = 15$ m

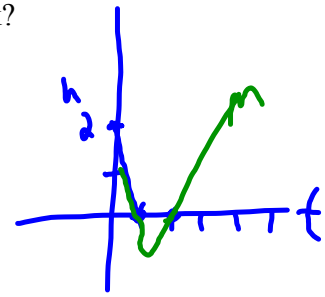
then $w = 50 - 2(15)$

$= 50 - 30$

$= 20$ m

the dimensions of the enclosure are 10 m by 30 m, **OR** 15 m by 20 m.

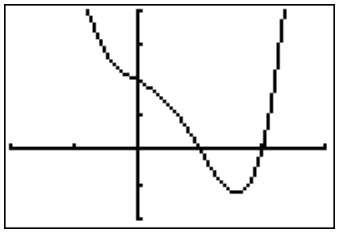
3. The function, $h = t^4 - 2t^3 - t + 2$, models the path of a seagull trying to catch fish, where h represents the seagull's height above the water in metres and t represents the time in seconds.
   a) At what height is the seagull when it first sees the fish?
   b) When does the seagull hit the water?
   c) At what time does the seagull leave the water with the fish in its beak?



a) $h(t) = t^4 - 2t^3 - t + 2$

if $t = 0$, $h(t) = 2$

∴ the sea gull is 2 m above the water.



b) when $h = 0$ or $h(t) = 0$

$0 = t^4 - 2t^3 - t + 2$

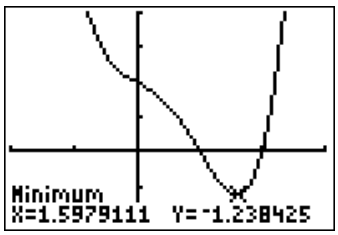
$= t^3(t-2) - 1(t-2)$

$= (t-2)(t^3 - 1)$

$t - 2 = 0$ ↳ $t^3 - 1 = 0$

$t = 2$ $t^3 = 1$

$t = \sqrt[3]{1}$



the seagull hits the water at 1 second (and leaves the water at 2 seconds).

4. Melissa is running a ski trip during the exam break.
   The bus holds 40 students and if she charges $250 per student the bus will be filled.
   For every $25 increase in the price she charges students, two fewer students will go on the trip.
   a) Write an equation to model Melissa's revenue.
   b) Determine the maximum revenue.
   c) How many students need to go on the trip for Melissa to earn $8800?

a) Revenue = price × number of tickets

$= p(-0.08p+60)$

$= -0.08p^2+60p$

$40-2\left(\frac{p-250}{25}\right)$

$= 40-\frac{2}{25}(p-250)$

$= 40-\frac{2}{25}p+20$

$= -\frac{2}{25}p+60$

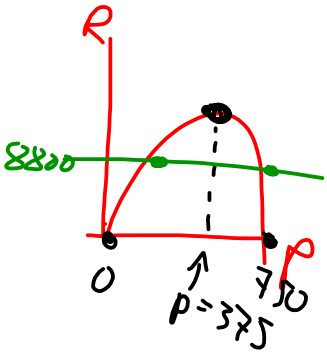
$= -0.08p+60$

b) $R = -0.08p^2+60p$

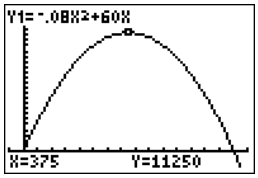
$= -0.08p(p-750)$

$p=0$ or $p=750$

$x=\frac{-b}{2a}$



the max. revenue would occur when ticket price is set at $375
(the max. revenue would be $11250, from 30 tickets being sold. 11250÷375=30)



c) $8800 = -0.08p^2+60p$

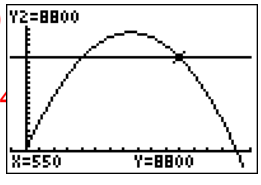
$0.08p^2-60p+8800=0$

$0.08(p^2-750p+11000)=0$

$0.08(p-550)(p-200)=0$

$p=550$ or $p=200$

to earn a revenue of $8800, ticket price must be set at $550
(resulting in only 16 tickets being sold 8800÷550=16)
or the ticket price must be set at $200
(resulting in 44 tickets needing to be sold 8800÷200=44
[What is the problem with this idea?]



Melissa is best off setting the price at $375

Answers:

1) The lawn will have a width of 10 m and the dimensions of the school will be 60 m by 40 m.
2) There are two possibilities: L=10 m by w=30 m AND L=15 m by w=20 m
3a) h = 2 m, b) t = 1 s, c) t = 2 s
4a) $R = -0.08p^2 + 60p$, where R is the revenue, and p is the price she charges, both in dollars
b) $11 250 (when price is $375)
c) 16 students earn her $8800 (when price is $550)