

Before we begin, are there any questions from last day's work? 4.1.2

**(Wednesday's quiz will be based on this material)**

## Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) determine the **EXACT** values of the sine, cosine, and tangent of the special angles  $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$  **and their multiples**
- b) understand and use the "CAST Rule" with the "raa" (related acute angle).

4.2.1 Trigonometric Ratios of Special Angles

Date: Oct-24/19

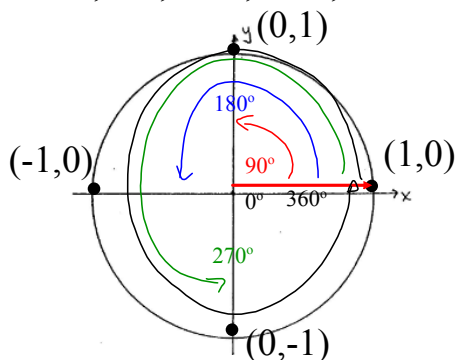
Recall:  $\sin\theta = \frac{y}{r}$        $\cos\theta = \frac{x}{r}$        $\tan\theta = \frac{y}{x}$

$y = r\sin\theta$        $x = r\cos\theta$

Note: Any point  $(x, y) = (r\cos\theta, r\sin\theta)$  and on a "unit circle",  $r = 1$

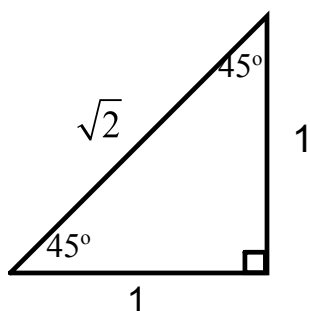
Any point  $(x, y) = (\cos\theta, \sin\theta)$  *alphabetical order*

A)  $\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

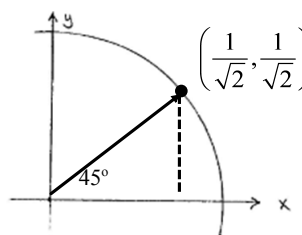


$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$   
 $\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$   
 $\cos 180^\circ = \frac{x}{r} = \frac{-1}{1} = -1$

B)  $\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

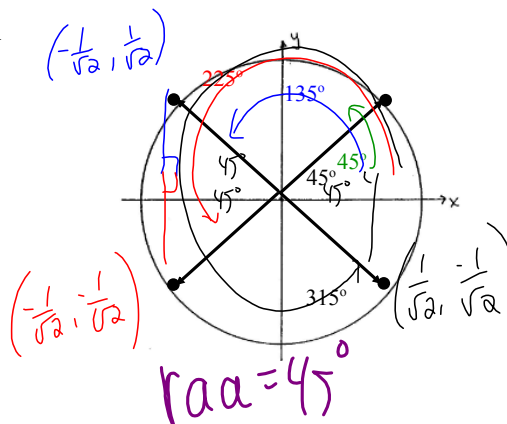


$\sin 45^\circ = \frac{1}{\sqrt{2}}$  or  $\frac{\sqrt{2}}{2}$   
 $\cos 45^\circ = \frac{1}{\sqrt{2}}$  or  $\frac{\sqrt{2}}{2}$   
 $\tan 45^\circ = \frac{1}{1} = 1$



Ex.  $\tan 315^\circ$

$= \frac{y}{x}$   
 $= \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$   
 $= -1$



$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

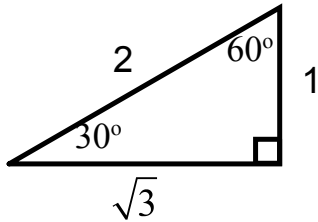
$$= \frac{\sqrt{2}}{\sqrt{4}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\sqrt{x} \sqrt{x}$$

$$= x$$

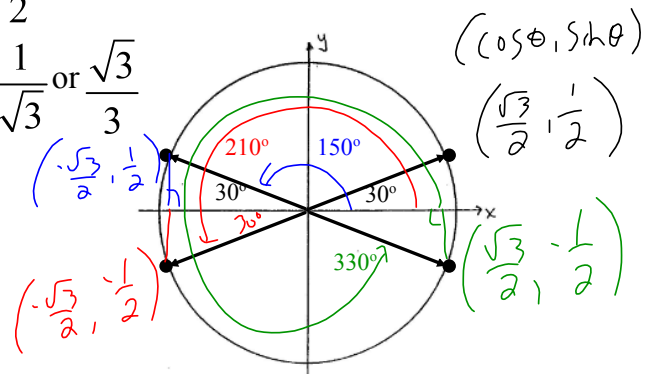
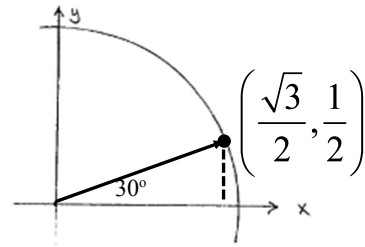
C)  $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$



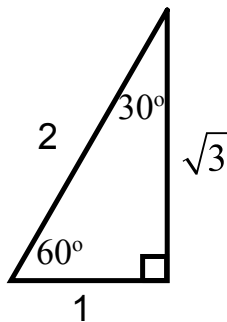
$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{3}$$



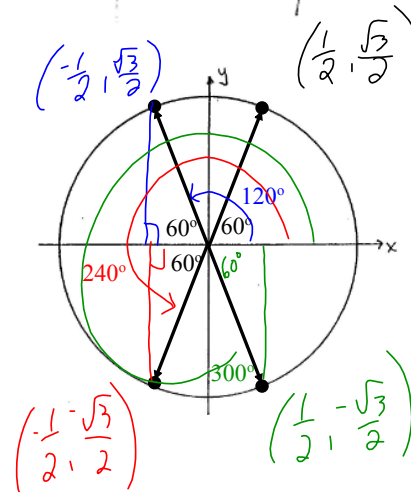
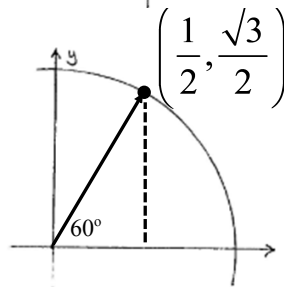
D)  $\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$



$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$



## 4.2.2 Trigonometric Ratios of Special Angles

Date: Oct. 23/19

$\theta$	$30^\circ$	$45^\circ$	$60^\circ$
$\sin \theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan \theta$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Memorize this Chart!

**The CAST rule:**

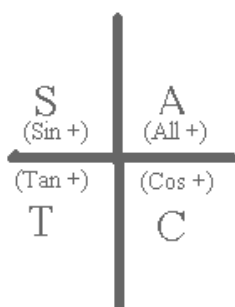
**The CAST RULE** is a memory aid which tells us the sign of the trig ratios in the various quadrants.

In the first quadrant **ALL** are positive. This is denoted using the letter **A**.

In the second quadrant **SINE** is positive. This is denoted by the letter **S** [ the other two ratios are negative ]

In the third quadrant **TANGENT** is positive. This is denoted by the letter **T** [ the other two ratios are negative ]

In the fourth quadrant **COSINE** is positive. This is denoted by the letter **C** [ the other two ratios are negative ]



$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{\sqrt{4}}$$

$$= \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{\sqrt{9}}$$

$$= \frac{\sqrt{3}}{3}$$

$$\sqrt{x} \cdot \sqrt{x} = x$$

4.2.3 Determining Trig Ratios of Special Angles

Method: Sketch, use the Related Acute Angle (raa), apply the CAST rule.

Evaluate:

1a)  $\sin 120^\circ = +\sin 60^\circ = \frac{\sqrt{3}}{2}$     b)  $\sin 225^\circ = -\sin 45^\circ = -\frac{1}{\sqrt{2}}$     c)  $\tan 300^\circ = -\tan 60^\circ = -\sqrt{3}$

2a)  $\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$     b)  $\sin 135^\circ = +\sin 45^\circ = +\frac{\sqrt{2}}{2}$     c)  $\tan 315^\circ = -\tan 45^\circ = -1$