

Date: Oct. 31/19

## Today's Learning Goal(s):

By the end of the class, I will be able to:

- use exponential functions to model exponential growth and decay.

Last day's work: pp. 251-253 #<sup>2b</sup>(1,2)ab, 3, 4ab, 5ab, 9  
(Optional Wkst 4.6 Extra Practice)  
(text questions at end of lesson)

*Order Change Spring 2018*  
pp. 251-253 #1, 5, 9, 10 [12 - 14]

*Copy all and leave some space.*

- Ex.1 You invest \$1000 at 8% /a compounded annually.  
 a) Model the amount of money as a growth function.  
 b) How much will you have after 20 years?

# of years	0	1	2	3			n
Amount							

- Ex.2 A superball loses 10% of its height after each bounce.  
 It was dropped from 12 *m*.  
 Model the bounce height with a decay function.
- Ex.3 A hockey card is purchased in 1990 for \$5.00.  
 The value increases by 6% each year.  
 Write an equation and determine it's value in 2011.
- Ex.4 In 1980 the population of the town of St. Albert, Alberta was 20 000.  
 If the town grows at a rate of 2% a year, what was the population in 2014?

## 4.7 Applications Involving Exponential Functions

$$A = P(1+i)^n$$

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Ex.1 You invest \$1000 at 8% *k* compounded annually.

- Model the amount of money as a growth function.
- How much will you have after 20 years?

# of years	0	1	2	3			n
Amount	1000	1080	1166.40				
		$1000(1.08)^1$	$1000(1.08)^2$	$1000(1.08)^3$			$1000(1.08)^n$

$$I = Prt$$

$$I_1 = 1000(0.08)(1)$$

$$= 80$$

$$I_2 = 1080(0.08)(1)$$

$$= 86.40$$

$$A = P + I$$

$$= 1000 + 80$$

$$= 1080$$

$$A_2 = 1080 + 86.40$$

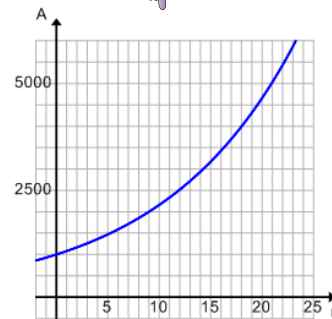
$$= 1166.40$$

$$A_1 = 1000(1.08)^1$$

$$= 1080$$

$$A_2 = 1080(1.08) \text{ or } A_2 = 1000(1.08)(1.08)$$

$$= 1166.40$$



$$A = P + I$$

$$= P + Prt$$

$$= P(1+rt)$$

$$= 1000(1+(0.08)(1))$$

$$= 1000(1.08)$$

b) if  $n=20$

$$A(n) = 1000(1.08)^n$$

$$A(20) = 1000(1.08)^{20}$$

$$\approx 4660.957$$

$$\approx \$4660.96$$

$$A = 1000(1.08)^n$$

Initial Amount

Growth Factor

Initial Amount 1000

Growth Rate 0.08

Growth Factor  $1+0.08$

$=1.08$

\$4660.96

Ex.2 A superball loses 10% of its height after each bounce.  
It was dropped from 12 *m*.

Model the bounce height with a decay function.

Initial Amount 12

Decay Rate 0.1

Decay Factor  $1 - 0.1$   
 $= 0.9$

$$H = 12(0.90)^n$$

Initial Amount      Decay Factor

$1 \pm r$

Each bounce is 90% of the previous bounce.

$$\begin{aligned}
 &100\% - 10\% \\
 &= 90\% \\
 &= 0.9
 \end{aligned}$$

The function  $f(x) = a(b^x)$  can be used as a model to solve problems involving exponential growth and decay.

$$f(x) = a(b^x)$$

Where  $a$  is the initial value,  
 $b$  is the growth factor and  
 $x$  is the number of compounding periods.

- Ex.3 A hockey card is purchased in 1990 for \$5.00.  
 The value increases by 6% each year.  
 a) Write an equation to represent the growth.  
 b) Determine the card's value in 2011.

Let  $V$  represent the value of the hockey card, in dollars.

Let  $n$  represent the number of years since 1990.

$$a) V(n) = 5(1.06)^n$$

or

$$V(n) = 5(1.06)^{n-1990}$$

\*change of statement

$$b) \text{ in } 2011$$

$$n = 2011 - 1990$$

$$= 21$$

$$V(21) = 5(1.06)^{21}$$

$$\doteq 16.998$$

$$\doteq \$17$$

\$17.00

Ex.4 In 1980 the population of the town of St. Albert, Alberta was 20 000.  
If the town grows at a rate of 2% a year, what was the population in 2014?

Let  $P$  represent the population.

Let  $n$  represent the number of years since 1980.

$$\begin{aligned}P(n) &= 20\,000(1 + 0.02)^n \\ &= 20\,000(1.02)^n\end{aligned}$$

$$\left\{ \begin{array}{l} 100\% + 2\% \\ = 102\% \\ = 1.02 \end{array} \right.$$

$$P(34) = 20\,000(1.02)^{34}$$

$$\doteq 39\,213.5$$

$$\doteq 39\,213$$

$$\begin{aligned}2014 - 1980 \\ = 34\end{aligned}$$

39 213

There are growth and decay applications that involve **doubling times** or **half-lives**. The formula can be altered to:

$$N(t) = N_o (2)^{\frac{t}{d}}$$

← total time  
← doubling time

$$N(t) = N_o \left( \frac{1}{2} \right)^{\frac{t}{d}}$$

← total time  
← amount of time to have **50%** left  
= **half-life**

Ex.5 A biology experiment starts with 1000 cells.  
 After 4 hours the count is estimated to be 256 000.  
 What is the doubling period for the cells?

Let  $C$  represent the number of cells.

Let  $d$  represent the doubling period, in hours.

$$C = 1000(2)^{\frac{t}{d}}$$

$$256\,000 = 1000(2)^{\frac{4}{d}}$$

$$\frac{256\,000}{1000} = 2^{\frac{4}{d}}$$

$$256 = 2^{\frac{4}{d}}$$

$$2^8 = 2^{\frac{4}{d}}$$

$$\begin{aligned} \therefore 8 &= \frac{4}{d} \\ d &= \frac{4}{8} \\ d &= \frac{1}{2} \end{aligned}$$

$\frac{1}{2}$  hour

the doubling period for cells is a  $\frac{1}{2}$  hour.



- Ex.5 A biology experiment starts with 1000 cells.  
After 4 hours the count is estimated to be 256 000.  
What is the doubling period for the cells?

$$256000 = 1000(2)^{\frac{4}{d}}$$

$$\frac{256000}{1000} = 2^{\frac{4}{d}}$$

$$256 = 2^{\frac{4}{d}}$$

$$2^{\square} (4 \div 8) = 1.4$$

$$2^{\square} (4 \div 1) = 16$$

$$2^{\square} (4 \div 0.25) = \text{Too big}$$

$$\boxed{2^{\square} (4 \div 0.5) = 256}$$

$\therefore$  my doubling time is  $\frac{1}{2}$  an hour.

**Are there any Homework Questions you would like to see on the board?**

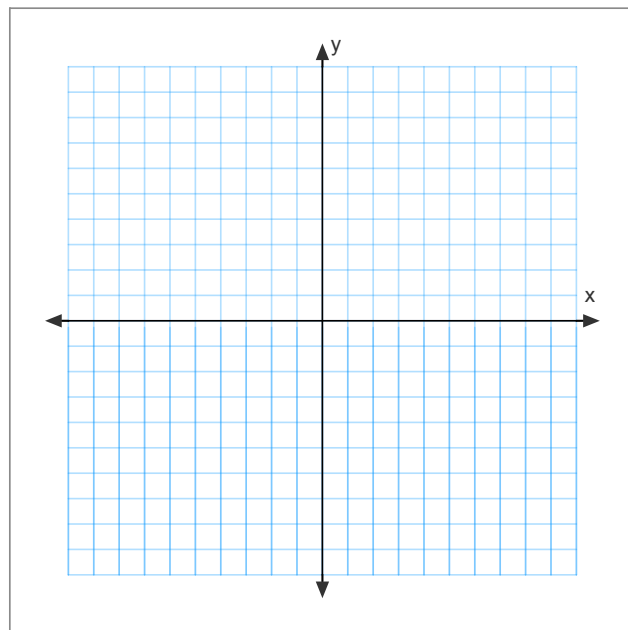
Last day's work: pp. 251-253 #(1,2)ab, 3, 4ab, 5ab, 9  
(*Oponal Wkst 4.6 Extra Pracce* )  
(text quesons on following screens)

Today's Homework Practice includes:

pp. 261-262 # 1 – 8

**SWYK NEXT CLASS**

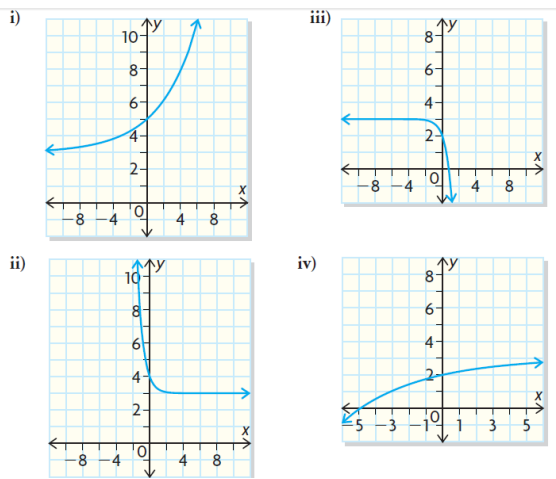
Hwk p. 252



- Each of the following are transformations of  $f(x) = 3^x$ . Describe each transformation.
  - $g(x) = 3^x + 3$
  - $g(x) = 3^{x+3}$
  - $g(x) = \frac{1}{3}(3^x)$
  - $g(x) = 3^{\frac{x}{3}}$
- For each transformation, state the base function and then describe the transformations in the order they could be applied.
  - $f(x) = -3(4^{x+1})$
  - $g(x) = 2\left(\frac{1}{2}\right)^{2x} + 3$
  - $h(x) = 7(0.5^{x-4}) - 1$
  - $k(x) = 5^{3x-6}$
- State the  $y$ -intercept, the equation of the asymptote, and the domain and range for each of the functions in questions 1 and 2.

**PRACTISING**

- Each of the following are transformations of  $h(x) = \left(\frac{1}{2}\right)^x$ . Use words to describe the sequence of transformations in each case.
  - $g(x) = -\left(\frac{1}{2}\right)^{2x}$
  - $g(x) = 5\left(\frac{1}{2}\right)^{-(x-3)}$
  - $g(x) = -4\left(\frac{1}{2}\right)^{3x+9} - 6$
- Let  $f(x) = 4^x$ . For each function that follows,
  - state the transformations that must be applied to  $f(x)$
  - state the  $y$ -intercept and the equation of the asymptote
  - sketch the new function
  - state the domain and range
  - $g(x) = 0.5f(-x) + 2$
  - $h(x) = -f(0.25x + 1) - 1$
  - $g(x) = -2f(2x - 6)$
  - $h(x) = f(-0.5x + 1)$
- Match the equation of the functions from the list to the appropriate graph at the top of the next page.
  - $f(x) = -\left(\frac{1}{4}\right)^{-x} + 3$
  - $y = \left(\frac{1}{4}\right)^x + 3$
  - $g(x) = -\left(\frac{5}{4}\right)^{-x} + 3$
  - $h(x) = 2\left(\frac{5}{4}\right)^x + 3$



- Each graph represents a transformation of the function  $f(x) = 2^x$ . Write an equation for each one.
  - 
  -

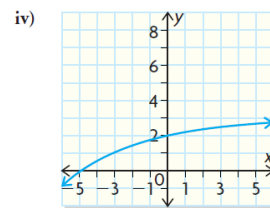
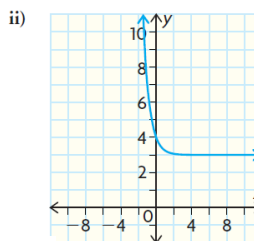
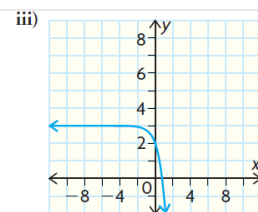
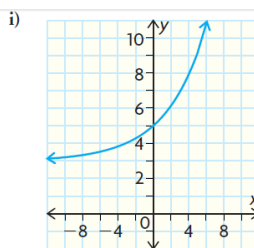
9. Match the equation of the functions from the list to the appropriate graph at the top of the next page.

a)  $f(x) = -\left(\frac{1}{4}\right)^{-x} + 3$

c)  $g(x) = -\left(\frac{5}{4}\right)^{-x} + 3$

b)  $y = \left(\frac{1}{4}\right)^x + 3$

d)  $h(x) = 2\left(\frac{5}{4}\right)^x + 3$



10. Each graph represents a transformation of the function  $f(x) = 2^x$ . Write an equation for each one.

