Date: ____________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) use the Sine Law to solve a triangle that is the **ambiguous** case.

7ace

Last day's work: pp. 300-301 #6 – 9ace, 10, 12 [15]

Review p. 304 #1 – 13

2

p. 300

7. For each trigonometric ratio in question 6, determine the smallest negative angle that has the same ratio.

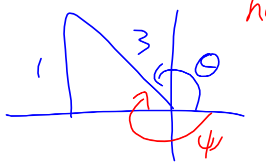
In Q II

a) $\sin \theta = \frac{1}{3}$

c) $\cos \theta = -\frac{1}{4}$

e) $\tan \theta = -1.1$

negative angle



$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

$\angle B \doteq 19.4$

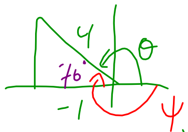
$\doteq 19^\circ$

$\therefore \theta = 180^\circ - 19^\circ$

$= 161^\circ$

$\psi = 161^\circ - 360^\circ$

$= -299^\circ$



$\theta = \cos^{-1}\left(-\frac{1}{4}\right)$

$\doteq 104.4$

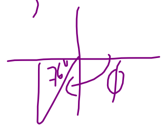
$\doteq 104^\circ$

$\psi \doteq 104 - 360^\circ$

$\doteq -256^\circ$

BuT

smaller



$\phi = -104^\circ$

× Ask Me Please

p. 301

10. Given each point $P(x, y)$ lying on the terminal arm of angle θ,

i) state the value of θ, using both a counterclockwise and a clockwise rotation

ii) determine the primary trigonometric ratios

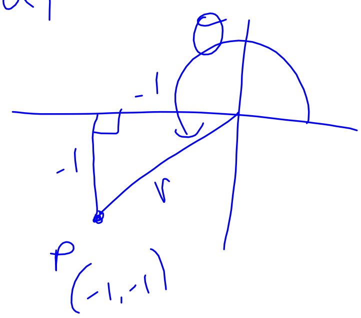
a) $P(-1, -1)$

b) $P(0, -1)$

c) $P(-1, 0)$

d) $P(1, 0)$

a)



$r^2 = x^2 + y^2$

$= (-1)^2 + (-1)^2$

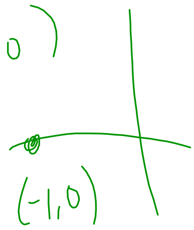
$= 2$

$r = \sqrt{2}$

$\sin\theta = \frac{-1}{\sqrt{2}}$ $\cos\theta = \frac{-1}{\sqrt{2}}$ $\tan\theta = \frac{-1}{-1}$

$= 1$

c) $P(-1, 0)$



$r^2 = (-1)^2 + (0)^2$

$= 1$

$r = 1$

$\sin\theta = \frac{0}{1}$ $\cos\theta = \frac{-1}{1}$ $\tan\theta = \frac{0}{1}$

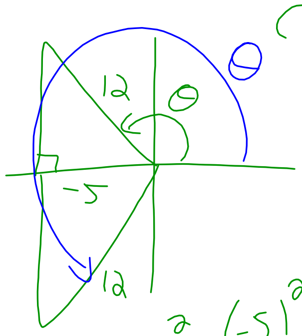
$= 0$ $= -1$ $= 0$

p. 301 **12.** Given $\cos\theta = -\frac{5}{12}$, where $0° \le \theta \le 360°$,

a) in which quadrant could the terminal arm of θ lie? → II or III

b) determine all possible primary trigonometric ratios for θ.

c) evaluate all possible values of θ to the nearest degree.



θ

$\cos\theta = \frac{x}{r}$

$= \frac{-5}{12}$

$y^2 = 12^2 - (-5)^2$

$= 144 - 25$

$= 119$

$y = \pm\sqrt{119}$

b) if $y = \sqrt{119}$

$\sin\theta = \frac{\sqrt{119}}{12}$ $\cos\theta = \frac{-5}{12}$

$\tan\theta = \frac{\sqrt{119}}{-5}$

b) if $y = -\sqrt{119}$

$\sin\theta = \frac{-\sqrt{119}}{12}$ $\cos\theta = \frac{-5}{12}$ $\tan\theta = \frac{-\sqrt{119}}{-5}$

$= \frac{\sqrt{119}}{5}$

c) if QII

$\theta = \cos^{-1}\left(\frac{-5}{12}\right)$

$\doteq 114.6$

$\doteq 115°$

$\therefore raa \doteq 180° - 115°$

$= 65°$

$\therefore$ in QIII

$\theta \doteq 180° + 65°$

$\doteq 245°$

p. 304 **2.** Determine the value of θ to the nearest degree if $0° \le \theta \le 90°$.

a) $\cot\theta = 0.8701$ c) $\csc\theta = 1.6406$

b) $\sec\theta = 4.1011$ d) $\sec\theta = 2.4312$

a) $\cot\theta = 0.8701$

$\frac{1}{\tan\theta} = \frac{0.8701}{1}$

$\tan\theta = \frac{1}{0.8701}$

$\theta = \tan^{-1}\left(\frac{1}{0.8701}\right)$

$\doteq 48.9$

$\doteq 49°$

b) $\sec\theta = 4.1011$

$\frac{1}{\cos\theta} = 4.1011$

$\cos\theta = \frac{1}{4.1011}$

$\theta = \cos^{-1}\left(\frac{1}{4.1011}\right)$

$\doteq 75.8$

$\doteq 76°$

5.6 The Sine Law

Date: NOV. 14/19

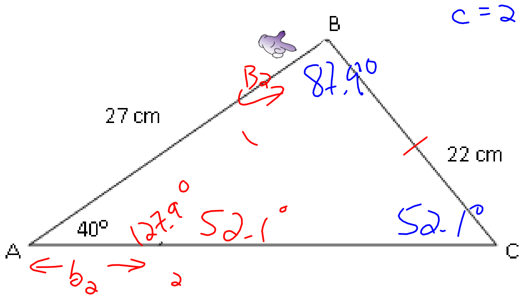
Recall: We use the Sine Law when we are have an "opposite pair".
The formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \textbf{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{A}{B} = \frac{C}{D}$$

$$AD = BC$$

Ex. 1 Consider ΔABC, $\angle A = 40°$, AB=27 cm, and BC = 22 cm. ***Make a sketch.***



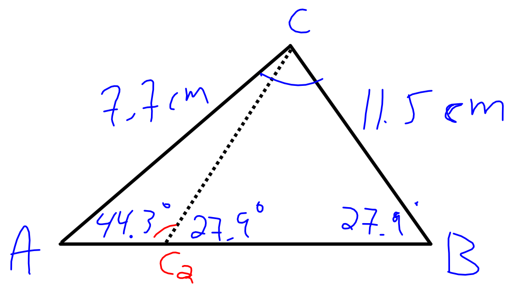
Note: There are 2 different ways to sketch ΔABC using this information.
This means there are two possible ways to solve this triangle.
This is the ambiguous case of the Sine Law.

$\angle C= 52.1°, \angle B= 87.9°$, b=34.2 cm

$\angle C= 127.9°, \angle B= 12.1°$, b=7.2 cm

$\angle C$	$\angle B$	b
$\frac{\sin C}{27} = \frac{\sin 40°}{22}$ $22\sin C = 27\sin 40°$ $\sin C = \frac{27\sin 40°}{22}$ $C = \sin^{-1}\left(\frac{27\sin 40°}{22}\right)$ $\doteq 52.08$ $\doteq 52.1°$	$B \doteq 180° - 40° - 52.1°$ $\doteq 87.9°$	$\frac{b}{\sin 87.9°} \doteq \frac{22}{\sin 40°}$ $b \doteq \sin 87.9° \times \frac{22}{\sin 40°}$ $\doteq 34.202$ $\doteq 34.20$
$\angle C_2$	$\angle B_2$	b_2
$\angle C_2 \doteq 180° - 52.1°$ $\doteq 127.9°$	$B_2 \doteq 180° - 40° - 127.9°$ $\doteq 12.1°$	$\frac{b_2}{\sin 12.1°} \doteq \frac{22}{\sin 40°}$ $b_2 \doteq \sin 12.1° \times \frac{22}{\sin 40°}$ $\doteq 7.174$ $\doteq 7.17$ cm

Ex. 2 Solve $\triangle ABC$, $\angle A = 44.3°$, $a = 11.5$ cm, and $b = 7.7$ cm. ***Make a sketch.***



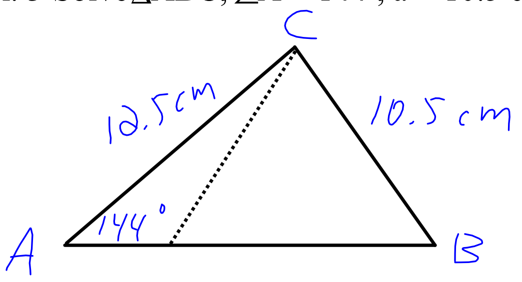
$\angle B = 27.9°$, $\angle C = 107.8°$, $c = 15.7$ cm

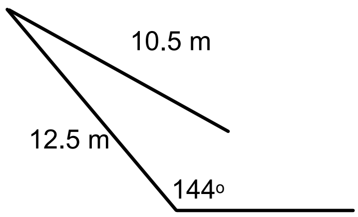
$\angle B = 152.1°$, $\angle C = -16.4°$

$\angle B$	$\angle C$	c
$\frac{\sin B}{7.7} = \frac{\sin 44.3°}{11.5}$ $\sin B = \frac{7.7 \sin 44.3°}{11.5}$ $B = \sin^{-1}\left(\frac{7.7 \sin 44.3°}{11.5}\right)$ $\doteq 27.88$ $\doteq 27.9°$	$C \doteq 180° - 44.3° - 27.9°$ $\doteq 107.8°$	$\frac{c}{\sin 107.8°} \doteq \frac{11.5}{\sin 44.3°}$ $c \doteq \sin 107.8° \times \frac{11.5}{\sin 44.3°}$ $\doteq 15.677$ $\doteq 15.68$ cm

$\angle B_2$	$\angle C_2$	
$B_2 \doteq 180° - 27.9°$ $\doteq 152.1°$	$C_2 \doteq 180° - 44.3° - 152.1$ $\doteq -16.4°$ $\therefore$ only 1 triangle is possible.	

Ex. 3 Solve ΔABC, $\angle A = 144°$, a = 10.5 cm, and b = 12.5 cm. ***Make a sketch.***





$\angle B = 44.4°, \angle C = -8.4°$

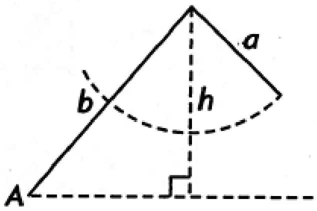
$\angle B$	$\angle C$	
$\frac{\sin B}{12.5} = \frac{\sin 144°}{10.5}$	$C \doteq 180° - 144° - 44.4°$	See Next Slide for New Summaries
$B = \sin^{-1}\left(12.5 \times \frac{\sin 144°}{10.5}\right)$	$\doteq -8.4°$	
$\doteq 44.40$	$\therefore$ No triangles possible	
$\doteq 44.4°$		

The ambiguous case arises in a SSA (side, side, angle) triangle. In this situation, depending on the size of the given angle and the lengths of the given sides, the sine law calculation may lead to 0, 1, or 2 solutions.

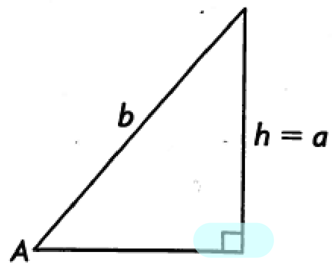
Need to Know

- In the ambiguous case, if $\angle A$, a, and b are given and $\angle A$ is acute, there are four cases to consider. In each case, the height of the triangle is $h = b \sin A$.

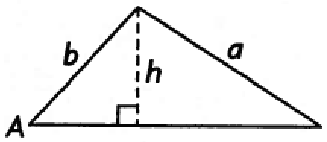
If $\angle A$ is acute and $a < h$, no triangle exists.



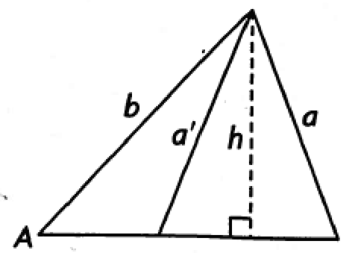
If $\angle A$ is acute and $a = h$, one right triangle exists.



If $\angle A$ is acute and $a > b$, one triangle exists.

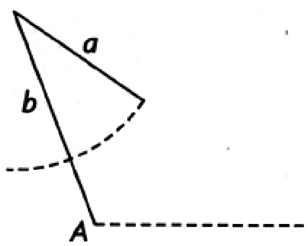


If $\angle A$ is acute and $h < a < b$, two triangles exist.

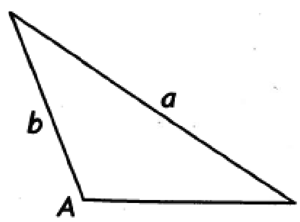


If $\angle A$, a, and b are given and $\angle A$ is obtuse, there are two cases to consider.

If $\angle A$ is obtuse and $a < b$ or $a = b$, no triangle exists.



If $\angle A$ is obtuse and $a > b$, one triangle exists.



Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 300-301 #6 – 9ace, 10, 12 [15]
Review p. 304 #1 – 13

Today's Homework Practice includes:

pp. 318-319 #1, 2, 3a, 4, 5ac, 7 [15,17]

Next screen?

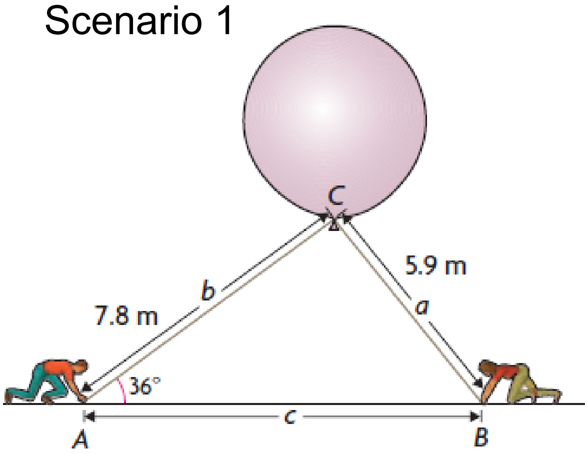
Ex. 4
Albert and Belle are part of a scientific team studying thunderclouds.
The team is about to launch a weather balloon into an active part of the cloud.
Albert's rope is 7.8 m long and makes an angle of 36^0 with the ground.
Belle's rope is 5.9 m long.
How far, to the nearest tenth of a metre, is Albert from Belle?

Sketch a diagram... where is Belle located?

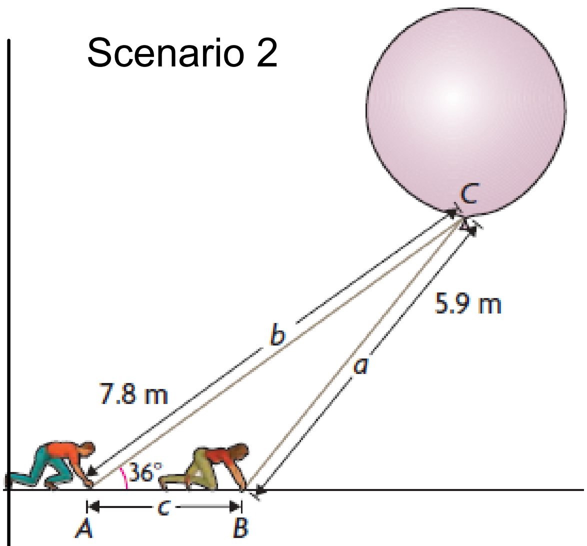
This is called an **ambiguous** case. We don't have enough information to determine exactly where Belle is located.

So.... we have to give both scenarios and give all possible answers.

Scenario 1



Scenario 2



Ex. 5 Determine the number of possible triangles if $a = 7.3$, $b = 14.6$ and $A=30°$.

Ex.6 A street-lamp can illuminate a distance up to 9 metres.
A person on the sidewalk is 51 metres from the top of the light.
From this point, the angle of elevation to the top of the light is 10^{0}.
What length of sidewalk is illuminated by the light?