

Date: Nov. 25/19

## Today's Learning Goal(s):

By the end of the class, I will be able to:

- a) prove trigonometric identities.

Last day's work: p. 310 #1 – 6

6  
3cda  
2d  
5  
4

p. 310 2. Simplify each expression.

a)  $(1 - \sin \alpha)(1 + \sin \alpha)$

c)  $\cos^2 \alpha + \sin^2 \alpha$

b)  $\frac{\tan \alpha}{\sin \alpha}$

d)  $\cot \alpha \sin \alpha$

$$= \frac{\cos \alpha}{\sin \alpha} \cdot \sin \alpha$$

$$= \cos \alpha$$

p. 310 3. Factor each expression.

a)  $1 - \cos^2 \theta$

b)  $\sin^2 \theta - \cos^2 \theta$

$$= (1 - \cos \theta)(1 + \cos \theta)$$

$$\cos \theta - \cos^2 \theta$$

let  $w = \cos \theta$

$$w - w^2$$

$$= w(1 - w)$$

$$= \cos \theta (1 - \cos \theta)$$

c)  $\sin^2 \theta - 2 \sin \theta + 1$

d)  $\cos \theta - \cos^2 \theta$

Let  $w = \sin \theta$

$$w^2 - 2w + 1$$

$$= (w - 1)(w - 1)$$

$$= (w - 1)^2$$

$$= (\sin \theta - 1)^2$$

- p. 310 4. Prove that  $\frac{\cos^2\phi}{1 - \sin\phi} = 1 + \sin\phi$ , where  $\sin\phi \neq 1$ , by expressing  $\cos^2\phi$  in terms of  $\sin\phi$ .

$$\begin{aligned} LS &= \frac{\cos^2\phi}{1 - \sin\phi} & RS &= 1 + \sin\phi \\ &= \frac{1 - \sin^2\phi}{1 - \sin\phi} & & \therefore LS = RS \\ &= \frac{(1 - \sin\phi)(1 + \sin\phi)}{1 - \sin\phi} & & \therefore QED. \\ &= 1 + \sin\phi \end{aligned}$$

- p. 310 5. Prove each identity. State any restrictions on the variables.

$$\begin{aligned} a) \frac{\sin x}{\tan x} &= \cos x & c) \frac{1}{\cos\alpha} + \tan\alpha &= \frac{1 + \sin\alpha}{\cos\alpha} \\ R: \tan x \neq 0 & & LS = \frac{\sin x}{\tan x} & LS = \frac{1}{\cos\alpha} + \tan\alpha \\ (\cos x, \sin x) & & = \frac{-\sin x \cdot \tan x}{\cos x} & = \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha} \\ \frac{\sin x}{\cos x} \neq 0 & & = -\sin x \cdot \frac{\sin x}{\cos x} & = \frac{1 + \sin\alpha}{\cos\alpha} \\ \therefore \sin x \neq 0 \text{ & } \cos x \neq 0 & & = -\sin^2 x & \\ x \neq 0, 180^\circ & & = \sin x \cdot \frac{\cos x}{\sin x} & = RS \\ x \neq 90^\circ, 270^\circ & & = \cos x & \therefore LS = RS \\ & & = RS & \therefore QED. \\ & & \therefore LS = RS & \end{aligned}$$

$$b) \frac{\tan\theta}{\cos\theta} = \frac{\sin\theta}{1 - \sin^2\theta}$$

$$\begin{aligned} LS &= \frac{\tan\theta}{\cos\theta} & RS &= \frac{\sin\theta}{1 - \sin^2\theta} & LS = 1 - \cos^2\theta & RS = \sin\theta \cos\theta \tan\theta \\ &= \frac{\sin\theta}{\cos\theta} \div \frac{1}{\cos\theta} & & & = \sin^2\theta & = \sin\theta \cos\theta \left( \frac{\sin\theta}{\cos\theta} \right) \\ &= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta} & & & & = \sin^2\theta \\ &= \frac{\sin\theta}{\cos^2\theta} & & & \therefore LS = RS & = \sin^2\theta \\ &= \frac{\sin\theta}{1 - \sin^2\theta} & & & \therefore QED & \end{aligned}$$

$$d) 1 - \cos^2\theta = \sin\theta \cos\theta \tan\theta$$

$$\begin{aligned} LS &= 1 - \cos^2\theta & RS &= \sin\theta \cos\theta \tan\theta \\ &= \sin^2\theta & & = \sin\theta \cos\theta \left( \frac{\sin\theta}{\cos\theta} \right) \\ & & & = \sin^2\theta \end{aligned}$$

$$\therefore LS = RS = \sin^2\theta$$

$\therefore QED$

- p. 310 6. Mark claimed that  $\frac{1}{\cot \theta} = \tan \theta$  is an identity. Marcia let  $\theta = 30^\circ$  and found that both sides of the equation worked out to  $\frac{1}{\sqrt{3}}$ . She said that this proves that the equation is an identity. Is Marcia's reasoning correct? Explain.

Marcia's reasoning is NOT correct, because she would ONLY be proving that

$$\frac{1}{\cot \theta} = \tan \theta \text{ when } \theta = 30^\circ, \text{ and not for all values of } \theta.$$

## 5.5 Trigonometric Identities (Day2)

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**Recall:**

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \begin{matrix} \sin^2 \theta = 1 - \cos^2 \theta \\ \cos^2 \theta = 1 - \sin^2 \theta \end{matrix}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\frac{\sin^2 \theta + \cos^2 \theta = 1}{\cancel{\cos^2 \theta} \quad \cancel{\cos^2 \theta} \quad \cancel{\cos^2 \theta}}$$

$$\frac{\sin^2 \theta}{\cancel{\sin^2 \theta}} + \frac{\cos^2 \theta}{\cancel{\sin^2 \theta}} = \frac{1}{\cancel{\sin^2 \theta}}$$

let  $a = \cos \theta$

Note: Sometimes using substitution can help simplify a question.

Ex. Simplify  $(1 - \cos \theta)(1 + \cos \theta)$       Change to  $(1 - a)(1 + a)$

$$\begin{aligned} & \cancel{1 + \cos \theta - \cos \theta - \cos^2 \theta} = 1 - a^2 \\ & = 1 - \cos^2 \theta \end{aligned}$$

**To Prove an Identity:**

- \* Separate the LS and RS, and work on them separately
- \* convert  $\tan$  and reciprocal ratios to  $\sin$  or  $\cos$
- \* apply the Pythagorean Identity, use common denominators & factor as required

Ex.1 Prove that  $\frac{\sin^2 x}{1 - \cos x} = 1 + \cos x$

$$\text{LS} = \frac{\sin^2 x}{1 - \cos x} \quad \text{RS} = 1 + \cos x$$

$$= \frac{1 - \cos^2 x}{1 - \cos x}$$

$$= \frac{(1 - \cos x)(1 + \cos x)}{1 - \cos x}$$

$$= 1 + \cos x$$

Laugh or Groan?

$$\frac{\sin(\text{gerine})}{\cos(\text{gerine})} = \text{_____}$$


$$\begin{aligned} 1 - x^2 \\ (1 - x)(1 + x) \end{aligned}$$

$$\therefore \text{LS} = \text{RS}$$

$$\therefore QED.$$

Ex.2 Prove that  $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{2}{\cos^2\theta}$

$$LS = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

$$RS = \frac{2}{\cos^2\theta}$$

$$= \frac{1}{1+\sin\theta} \left( \frac{1-\sin\theta}{1-\sin\theta} \right) + \frac{1}{1-\sin\theta} \left( \frac{1+\sin\theta}{1+\sin\theta} \right)$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)} + \frac{1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{2}{(1+\sin\theta)(1-\sin\theta)}$$

$$\therefore LS = RS$$

$\therefore$  QED

Are there any Questions from the Assigned Practice that you would like to see on the board?

Last day's work: p. 310 #1 – 6

Today's Homework Practice includes:

pp. 310-311 #8, 10 – 12 [14]

Worksheet a – j (*online*)

Hints? (*If needed*)

8. Prove each identity. State any restrictions on the variables.

a)  $\frac{\sin^2 \phi}{1 - \cos \phi} = 1 + \cos \phi$

b)  $\frac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$

c)  $\cos^2 x = (1 - \sin x)(1 + \sin x)$

d)  $\sin^2 \theta + 2 \cos^2 \theta - 1 = \cos^2 \theta$

e)  $\sin^4 \alpha - \cos^4 \alpha = \sin^2 \alpha - \cos^2 \alpha$

f)  $\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}$

b) Solution 1: LS - use the quotient identity,  
and then simplify the fraction.

Solution 2: LS - use version 2 of the Pythagorean identity.

d) LS - sub in  $\sin^2 \theta$

e) LS - factor the difference of squares

f) LS - add the fractions then sub for  $\tan \theta$

12. Prove each identity. State any restrictions on the variables.

T a)  $\frac{\sin^2 \theta + 2 \cos \theta - 1}{\sin^2 \theta + 3 \cos \theta - 3} = \frac{\cos^2 \theta + \cos \theta}{-\sin^2 \theta}$

b)  $\sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \frac{2 \sin^2 \alpha - 2 \sin^4 \alpha - 1}{1 - \sin^2 \alpha}$

a) sub for  $\sin^2 \theta$  on both sides,  
then factor and divide.

b) LS - sub for  $\sin^2 \theta$  and use the quotient rule,  
then add the fractions.

## Extending

14. a) Which equations are not identities? Justify your answers.

b) For those equations that are identities, state any restrictions on the variables.

i)  $(1 - \cos^2 x)(1 - \tan^2 x) = \frac{\sin^2 x - 2 \sin^4 x}{1 - \sin^2 x}$

ii)  $1 - 2 \cos^2 \phi = \sin^4 \phi - \cos^4 \phi$

iii)  $\frac{\sin \theta \tan \theta}{\sin \theta + \tan \theta} = \sin \theta \tan \theta$

iv)  $\frac{1 + 2 \sin \beta \cos \beta}{\sin \beta + \cos \beta} = \sin \beta + \cos \beta$

v)  $\frac{1 - \cos \beta}{\sin \beta} = \frac{\sin \beta}{1 + \cos \beta}$

vi)  $\frac{\sin x}{1 + \cos x} = \csc x - \cot x$

iv) LS - sub  $\sin^2 \theta + \cos^2 \theta$  in for 1,  
and then factor and divide.

v) LS - multiply top and bottom by  $1 + \cos \beta$

vi) RS - put in terms of  $\sin x$  and  $\cos x$  and then see above.