Date: ___________________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) prove trigonometric identities.

Last day's work: pp. 310-311 #8, 10 – 12 [14]
Worksheet a – j (*online*)

12b a
8b d
14

c b
i g
j

p. 310 **8.** Prove each identity. State any restrictions on the variables.

b) $\dfrac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$ let $\alpha = \theta$

$LS = \dfrac{\tan^2 \theta}{1 + \tan^2 \theta}$

$= \dfrac{\tan^2 \theta}{\sec^2 \theta}$

$= \dfrac{\sin^2 \theta}{\cos^2 \theta} \div \sec^2 \theta$

$= \dfrac{\sin^2 \theta}{\cos^2 \theta} \div \dfrac{1}{\cos^2 \theta}$

$= \dfrac{\sin^2 \theta}{\cancel{\cos^2} \theta} \times \dfrac{\cancel{\cos^2} \theta}{1}$

$= \sin^2 \theta$

$= RS$

$\therefore LS = RS$

$\therefore$ QED.

d) $\sin^2 \theta + 2 \cos^2 \theta - 1 = \cos^2 \theta$

$LS = \sin^2 \theta + 2\cos^2 \theta - 1$ $\quad RS = \cos^2 \theta$

$= \underline{\underline{1}} - \underline{\cos^2 \theta} + 2\underline{\cos^2 \theta} - \underline{\underline{1}}$

$= \cos^2 \theta$

$\therefore LS = RS$ No Restriction

$\therefore$ QED.

p. 311

12. Prove each identity. State any restrictions on the variables.

a) $\dfrac{\sin^2\theta + 2\cos\theta - 1}{\sin^2\theta + 3\cos\theta - 3} = \dfrac{\cos^2\theta + \cos\theta}{-\sin^2\theta}$

$$LS = \frac{1-\cos^2\theta + 2\cos\theta - 1}{1-\cos^2\theta + 3\cos\theta - 3}$$

$$= \frac{-\cos^2\theta + 2\cos\theta}{-\cos^2\theta + 3\cos\theta - 2}$$

$$= \frac{-\cos\theta(\cos\theta - 2)}{-1(\cos^2\theta - 3\cos\theta + 2)}$$

$$= \frac{-\cos\theta(\cancel{\cos\theta - 2})}{-1(\cancel{\cos\theta - 2})(\cos\theta - 1)}$$

$$= \frac{\cos\theta}{\cos\theta - 1}$$

$$RS = \frac{\cos\theta(\cos\theta + 1)}{-(1-\cos^2\theta)}$$

$$= \frac{\cos\theta(\cos\theta+1)}{-1+\cos^2\theta}$$

$$= \frac{\cos\theta(\cos\theta+1)}{\cos^2\theta - 1}$$

$$= \frac{\cos\theta(\cancel{\cos\theta+1})}{(\cancel{\cos\theta+1})(\cos\theta-1)}$$

$$= \frac{\cos\theta}{\cos\theta - 1}$$

$\therefore LS = RS$

$\therefore$ QED!!

b) $\sin^2\alpha - \cos^2\alpha - \tan^2\alpha = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1 - \sin^2\alpha}$

$$LS = \sin^2\theta - \cos^2\theta - \tan^2\theta$$

$$= 1 - \cos^2\theta - \cos^2\theta - \tan^2\theta$$

$$= 1 - 2\cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta} - 2\cos^2\theta\left(\frac{\cos^2\theta}{\cos^2\theta}\right) - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta - 2\cos^4\theta - \sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta - 2(\cos^2\theta)(\cos^2\theta) - \sin^2\theta}{1-\sin^2\theta}$$

$$= \frac{1 - 2\sin^2\theta - 2(1-\sin^2\theta)(1-\sin^2\theta)}{1-\sin^2\theta}$$

$$= \frac{1 - 2\sin^2\theta - 2(1 - 2\sin^2\theta + \sin^4\theta)}{1-\sin^2\theta}$$

$$= \frac{1 - 2\sin^2\theta - 2 + 4\sin^2\theta - 2\sin^4\theta}{1-\sin^2\theta}$$

$$= \frac{-1 + 2\sin^2\theta - 2\sin^4\theta}{1-\sin^2\theta}$$

$$RS = \frac{2\sin^2\theta - 2\sin^4\theta - 1}{1-\sin^2\theta}$$

$$= \frac{-2\sin^4\theta + 2\sin^2\theta - 1}{(1-\sin\theta)(1+\sin\theta)}$$

$\therefore LS = RS$

$\therefore$ QED.

p. 310 An alternate proof to the same identy.

12 **b)** $\sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \dfrac{2\sin^2 \alpha - 2\sin^4 \alpha - 1}{1 - \sin^2 \alpha}$

$LS = \sin^2\alpha - \cos^2\alpha - \tan^2\alpha$

$= \sin^2\alpha - \cos^2\alpha - \dfrac{\sin^2\alpha}{\cos^2\alpha}$

$= \dfrac{\sin^2\alpha\cos^2\alpha}{\cos^2\alpha} - \cos^2\alpha\left(\dfrac{\cos^2\alpha}{\cos^2\alpha}\right)$

$= \dfrac{\sin^2\alpha\cos^2\alpha - \cos^4\alpha - \sin^2\alpha}{\cos^2\alpha}$

$= \dfrac{\sin^2\alpha(1-\sin^2\alpha) - (1-\sin^2\alpha)(1-\sin^2\alpha) - \sin^2\alpha}{1-\sin^2\alpha}$

$= \dfrac{\sin^2\alpha - \sin^4\alpha - (1 - 2\sin^2\alpha + \sin^4\alpha) - \sin^2\alpha}{1-\sin^2\alpha}$

$= \dfrac{\sin^2\alpha - \sin^4\alpha - 1 + 2\sin^2\alpha - \sin^4\alpha - \sin^2\alpha}{1-\sin^2\alpha}$

$= \dfrac{-2\sin^4\alpha + 2\sin^2\alpha - 1}{1-\sin^2\alpha}$

$RS = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

$= \dfrac{-2\sin^4\alpha + 2\sin^2\alpha - 1}{1-\sin^2\alpha}$

$\therefore LS = RS$

$\therefore$ QED!!

p. 310 **Another alternate proof to the same identy.**

12 **b)** $\sin^2\alpha - \cos^2\alpha - \tan^2\alpha = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

$LS = \sin^2\alpha - \cos^2\alpha - \tan^2\alpha$

$= 1 - \cos^2\alpha - \cos^2\alpha - \tan^2\alpha$

$= 1 - 2\cos^2\alpha - \dfrac{\sin^2\alpha}{\cos^2\alpha}$

$= \dfrac{\cos^2\alpha}{\cos^2\alpha} - \dfrac{2\cos^2\alpha\cos^2\alpha}{\cos^2\alpha} - \dfrac{\sin^2\alpha}{\cos^2\alpha}$

$= \dfrac{\cos^2\alpha - 2\cos^4\alpha - (1-\cos^2\alpha)}{\cos^2\alpha}$

$= \dfrac{\cos^2\alpha - 2\cos^4\alpha - 1 + \cos^2\alpha}{\cos^2\alpha}$

$= \dfrac{-2\cos^4\alpha + 2\cos^2\alpha - 1}{\cos^2\alpha}$

$RS = \dfrac{2\sin^2\alpha - 2\sin^4\alpha - 1}{1-\sin^2\alpha}$

$= \dfrac{2(1-\cos^2\alpha) - 2(1-\cos^2\alpha)(1-\cos^2\alpha) - 1}{1-\sin^2\alpha}$

$= \dfrac{2 - 2\cos^2\alpha - 2(1 - 2\cos^2\alpha + \cos^4\alpha) - 1}{1-\sin^2\alpha}$

$= \dfrac{\cancel{2} - 2\cos^2\alpha - \cancel{2} + 4\cos^2\alpha - 2\cos^4\alpha - 1}{1-\sin^2\alpha}$

$= \dfrac{2\cos^2\alpha - 2\cos^4\alpha - 1}{\cos^2\alpha}$

$= \dfrac{-2\cos^4\alpha + 2\cos^2\alpha - 1}{\cos^2\alpha}$

$\therefore LS = RS$

$\therefore$ QED.

5.5 Trigonometric Identities (Day3)

Date: ________________________

Recall:

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 310-311 #8, 10 – 12 [14]
Worksheet a – j (*online*)

Study for the Trig Identities Quiz!

Today's Homework Practice includes:
p. 339 #6, 7
p. 340 #4

8. Prove each identity. State any restrictions on the variables.

a) $\dfrac{\sin^2 \phi}{1 - \cos \phi} = 1 + \cos \phi$

b) $\dfrac{\tan^2 \alpha}{1 + \tan^2 \alpha} = \sin^2 \alpha$

c) $\cos^2 x = (1 - \sin x)(1 + \sin x)$

d) $\sin^2 \theta + 2\cos^2 \theta - 1 = \cos^2 \theta$

e) $\sin^4 \alpha - \cos^4 \alpha = \sin^2 \alpha - \cos^2 \alpha$

f) $\tan \theta + \dfrac{1}{\tan \theta} = \dfrac{1}{\sin \theta \cos \theta}$

b) Solution 1: LS - use the quotient identity, and then simplify the fraction.
Solution 2: LS - use version 2 of the Pythagorean identity.

d) LS - sub in $\sin^2\theta$

e) LS - factor the difference of squares

f) LS - add the fractions then sub for $\tan\theta$

12. Prove each identity. State any restrictions on the variables.

T

a) $\dfrac{\sin^2 \theta + 2\cos \theta - 1}{\sin^2 \theta + 3\cos \theta - 3} = \dfrac{\cos^2 \theta + \cos \theta}{-\sin^2 \theta}$

b) $\sin^2 \alpha - \cos^2 \alpha - \tan^2 \alpha = \dfrac{2\sin^2 \alpha - 2\sin^4 \alpha - 1}{1 - \sin^2 \alpha}$

a) sub for $\sin^2\theta$ on both sides, then factor and divide.

b) LS - sub for $\sin^2\theta$ and use the quotient rule, then add the fractions.

Extending

14. a) Which equations are not identities? Justify your answers.

b) For those equations that are identities, state any restrictions on the variables.

i) $(1 - \cos^2 x)(1 - \tan^2 x) = \dfrac{\sin^2 x - 2\sin^4 x}{1 - \sin^2 x}$

ii) $1 - 2\cos^2 \phi = \sin^4 \phi - \cos^4 \phi$

iii) $\dfrac{\sin \theta \tan \theta}{\sin \theta + \tan \theta} = \sin \theta \tan \theta$

iv) $\dfrac{1 + 2\sin \beta \cos \beta}{\sin \beta + \cos \beta} = \sin \beta + \cos \beta$

v) $\dfrac{1 - \cos \beta}{\sin \beta} = \dfrac{\sin \beta}{1 + \cos \beta}$

vi) $\dfrac{\sin x}{1 + \cos x} = \csc x - \cot x$

iv) LS - sub $\sin^2 \theta + \cos^2 \theta$ in for 1, and then factor and divide.

v) LS - multiply top and bottom by $1 + \cos\beta$

vi) RS - put in terms of sinx and cosx and then see above.

From the Trigonometic Identities Worksheet (online)

Prove each of the following idenes:

a) $\tan A + \cot A = \sec A \cdot \csc A$

b) $\cot^2 a = \cos^2 a + (\cot a \cdot \cos a)^2$

$LS = \cot^2 a$

$RS = \cos^2 a + (\cot a \cdot \cos a)^2$

$= \cos^2 a + \cot^2 a \cdot \cos^2 a$

$= \cos^2 a(1 + \cot^2 a)$

$= \cos^2 a(\csc^2 a)$

$= \cos^2 a\left(\frac{1}{\sin^2 a}\right)$

$= \frac{\cos^2 a}{\sin^2 a}$

$= \cot^2 a$

$\therefore LS = RS$

$\therefore$ Q.E.D.

c) $\frac{1}{sec^2\theta} = sin^2\theta \cdot cos^2\theta + cos^4\theta$

d) $\cot\theta \cdot \sec\theta = \csc\theta$

$LS = \frac{1}{\sec^2\theta}$

$= \cos^2\theta$

$RS = \sin^2\theta \cdot \cos^2\theta + \cos^4\theta$

$= \cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= \cos^2\theta(1)$

$x^2y^2 + y^4$

$= y^2(x^2 + y^2)$

$\therefore LS = RS$

$\therefore$ QED

e) $sec^2\theta + csc^2\theta = \frac{1}{sin^2\theta \cdot cos^2\theta}$

f) $\frac{1+tan^2\theta}{1+cot^2\theta} = tan^2\theta$

g) $\frac{sec^2\alpha - cos^2\alpha}{tan^2\alpha} = 1 + cos^2\alpha$

h) sin A · cos A · tan A = 1 – cos^2 A

$LS = \frac{sec^2\theta - cos^2\theta}{tan^2\theta}$

$= \frac{1 + tan^2\theta - cos^2\theta}{tan^2\theta}$

$= \left(1 + \frac{sin^2\theta}{cos^2\theta} - cos^2\theta\right) \div tan^2\theta$

$= \left(\frac{cos^2\theta}{cos^2\theta} + \frac{sin^2\theta}{cos^2\theta} - cos^2\theta\right) \div \frac{sin^2\theta}{cos^2\theta}$

$= \left(\frac{1}{cos^2\theta} - cos^2\theta\right) \times \frac{cos^2\theta}{sin^2\theta}$

$= \left(\frac{1}{\cancel{cos^2\theta}} - \frac{cos^4\theta}{\cancel{cos^2\theta}}\right) \times \frac{\cancel{cos^2\theta}}{sin^2\theta}$

$= (1 - cos^4\theta) \times \frac{1}{sin^2\theta}$

$= (1 - cos^2\theta)(1 + cos^2\theta) \times \frac{1}{sin^2\theta}$

$= (\cancel{sin^2\theta})(1 + cos^2\theta) \times \frac{1}{\cancel{sin^2\theta}}$

$= 1 + cos^2\theta$

$= RS$

i) $\frac{csc^2\rho - sin^2\rho}{csc^2\rho\cdot(2-cos^2\rho)} = cos^2\rho$

i) $LS = \frac{csc^2\rho - sin^2\rho}{csc^2\rho\,(2-cos^2\rho)}$ $RS = cos^2\rho$

$= \frac{\frac{1}{sin^2\rho} - sin^2\rho}{\frac{1}{sin^2\rho}(1+1-cos^2\rho)}$

$= \frac{\frac{1}{sin^2\rho} - sin^2\rho\left(\frac{sin^2\rho}{sin^2\rho}\right)}{\frac{1}{sin^2\rho}(1+sin^2\rho)}$

$= \frac{\frac{1}{sin^2\rho} - \frac{sin^4\rho}{sin^2\rho}}{\frac{1+sin^2\rho}{sin^2\rho}}$

$= \frac{1-sin^4\rho}{sin^2\rho} \div \frac{1+sin^2\rho}{sin^2\rho}$

$= \frac{(1-sin^2\rho)(1+sin^2\rho)}{sin^2\rho} \times \frac{sin^2\rho}{1+sin^2\rho}$

$= 1-sin^2\rho$

$= cos^2\rho$

$= RS$

$\because LS = RS$

$\therefore$ QED.

j) $(1 + \tan^2 x)(1 - \cos^2 x) = \tan^2 x$

j) $LS = (1+tan^2x)(1-cos^2x)$ $RS = tan^2x$

$= (sec^2x)(sin^2x)$

$= \frac{1}{cos^2x} \cdot sin^2x$ $\because LS = RS$

$= tan^2x$ $\therefore$ QED