Date: ______________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) understand the pattern in Pascal's triangle.
b) use Pascal's triangle to expand binomials efficiently.

Return and Correct CheckPoint 7.1

CheckPoint 7.2 is today!

Last day's work: pp. 459-461 #(1 – 6)ace, 9, 11, 13 [16,18]
(G. Series) (Mon, Dec. 16)
be

p. 459 **1.** Calculate the sum of the first seven terms of each geometric series.

a) $6 + 18 + 54 + \dots$ **c)** $8 - 24 + 72 - \dots$

b) $100 + 50 + 25 + \dots$ **d)** $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \dots$

c) g. series

$a = 8$

$r = -3$

$n = 7$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{8((-3)^7 - 1)}{(-3) - 1}$$

$$= \frac{8((-3)^7 - 1)}{-4}$$

$$= 4376$$

p. 460

6. Calculate the sum of each geometric series.

a) $1 + 6 + 36 + \ldots + 279\,936$

b) $960 + 480 + 240 + \ldots + 15$

c) $17 - 51 + 153 - \ldots - 334\,611$

d) $24\,000 + 3600 + 540 + \ldots + 1.8225$

e) $-6 + 24 - 96 + \ldots + 98\,304$

g ser.

$a = -6$

$r_1 = \frac{24}{-6}$ $\quad r_2 = \frac{-96}{24}$

$r = -4$ $\quad = -4$

$t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{(-6)((-4)^n - 1)}{-4-1}$

$= \frac{-6((-4)^n - 1)}{-5}$

$t_n = 98\,304$

$98\,304 = -6(-4)^{n-1}$

$-16\,384 = (-4)^{n-1}$

$\therefore (-4)^7 = -16\,384$

$\therefore 7 = n-1$

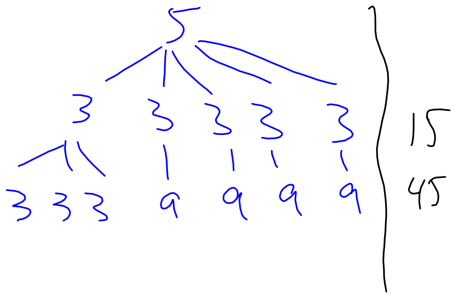
$\boxed{\therefore n = 8}$

$\therefore S_8 = \frac{-6((-4)^8 - 1)}{-5}$

$= 78\,642$

p. 461

11. A large company has a phone tree to contact its employees in case of an emergency factory shutdown. Each of the five senior managers calls three employees, who each call three other employees, and so on. If the tree consists of seven levels, how many employees does the company have?



$S_n = 5 + 15 + 45$

$\therefore a = 5, \; r = 3 \quad n = 7$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

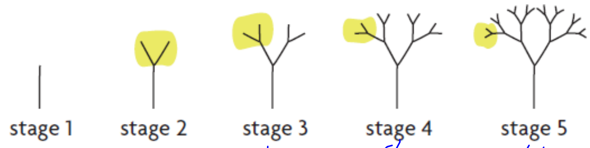
$$S_7 = \frac{5(3^7 - 1)}{3 - 1}$$

$$= \frac{5(2187 - 1)}{2}$$

$$= 5465$$

Below p. 460 #9 p. 461 #13

p. 460 **9** **A** A simple fractal tree grows in stages. At each new stage, two new line segments branch out from each segment at the top of the tree. The first five stages are shown. How many line segments need to be drawn to create stage 20?



New Segments 1 2 4 8 16

$r = 2$

g.Series

$a = 1$

$r = 2$

$n = 20$

$t_n = ar^{n-1}$

$t_{20} = 1(2)^{19}$

$=$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{20} = \frac{1((2)^{20} - 1)}{2 - 1}$

$= \frac{(2^{20} - 1)}{1}$

$= 1\,048\,575$

p. 461 **13.** **T** A cereal company attempts to promote its product by placing certificates for a cash prize in selected boxes. The company wants to come up with a number of prizes that satisfy all of these conditions:

a) The total of the prizes is at most \$2 000 000.

b) Each prize is in whole dollars (no cents).

c) When the prizes are arranged from least to greatest, each prize is a constant integral multiple of the next smaller prize and is
- more than double the next smaller prize $r > 2$
- less than 10 times the next smaller prize $r < 10$

$\therefore 2 < r < 10$

Determine a set of prizes that satisfies these conditions.

$S_n = 2\,000\,000$

try $a = \$100$

$r = 3$

$\therefore 100, 300, 900, \ldots$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$2\,000\,000 = \frac{100(3^n - 1)}{3 - 1}$

$4\,000\,000 = 100(3^n - 1)$

$40\,000 = 3^n - 1$

$40\,001 = 3^n$

$3^9 = 19\,683$

$3^{10} = 59\,049$

$\therefore n = 9$ max

$S_9 = \frac{100(3^9 - 1)}{3 - 1}$

$= 984\,100$

which does not exceed 2 000 000

$\therefore$ there are many possible answers.

p. 461 **16.** In a geometric series, $t_1 = 23$, $t_3 = 92$, and the sum of all of the terms of the series is 62 813. How many terms are in the series?

g.series

$a = 23$

$r = ?$

$n = ?$

$t_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_n = 62813$

$\left(\frac{t_3}{t_1}\right) \quad \frac{ar^2}{a} = \frac{92}{23}$

$r^2 = 4$

$\therefore r = \pm 2$

if $r = 2$

$S_n = \frac{23(2^n - 1)}{2-1}$

$= \frac{23(2^n - 1)}{1}$

$62813 = 23(2^n - 1)$

$2731 = 2^n - 1$

$2732 = 2^n$

$2^{11} = 2048$, $2^{12} = 4096$

$\therefore$ No $n \in \mathbb{N}$ exists

if $r = -2$

$S_n = \frac{23((-2)^n - 1)}{-2-1}$

$62813 = \frac{23((-2)^n - 1)}{-3}$

$\frac{-188439}{23} = (-2)^n - 1$

$-8193 = (-2)^n - 1$

$-8192 = (-2)^n$

$\therefore n = 13$

$\therefore$ there are 13 terms in the series.

7.7 Pascal's Triangle and Binomial Expansions

Date: Jan. 6/20

Ex.1 Expand and simplify each of the following:

$(a + b)^1$

$= a + b$

$(a + b)^2$

$= a^2 + 2ab + b^2$

$(a + b)^3$

$= (a + b)(a^2 + 2ab + b^2)$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$

$(a + b)^4$

$= (a + b)(a^3 \quad)$

$= a^4 + 3a^3b + 3a^2b^2 + ab^3 + a^3b + 3a^2b^2 + 3ab^3 + b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Before continuing, let's explore Pascal's Triangle!

Click on the paperclip to learn about Pascal's Triangle.

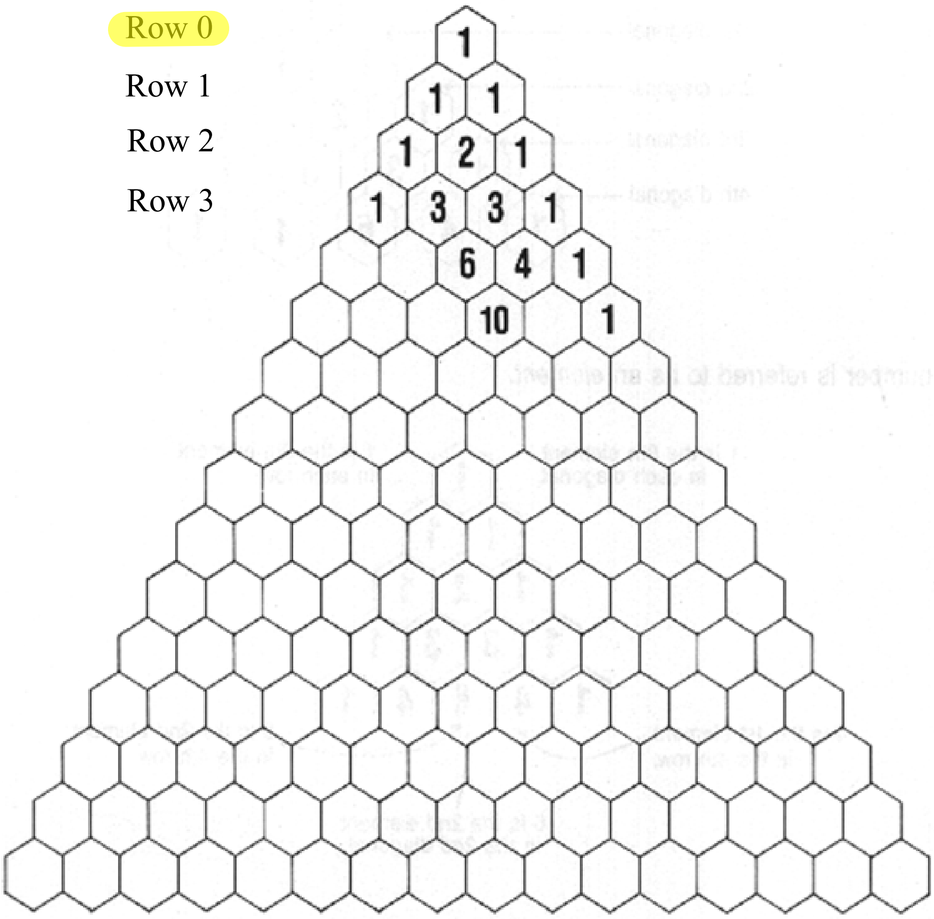
Pascal's Triangle

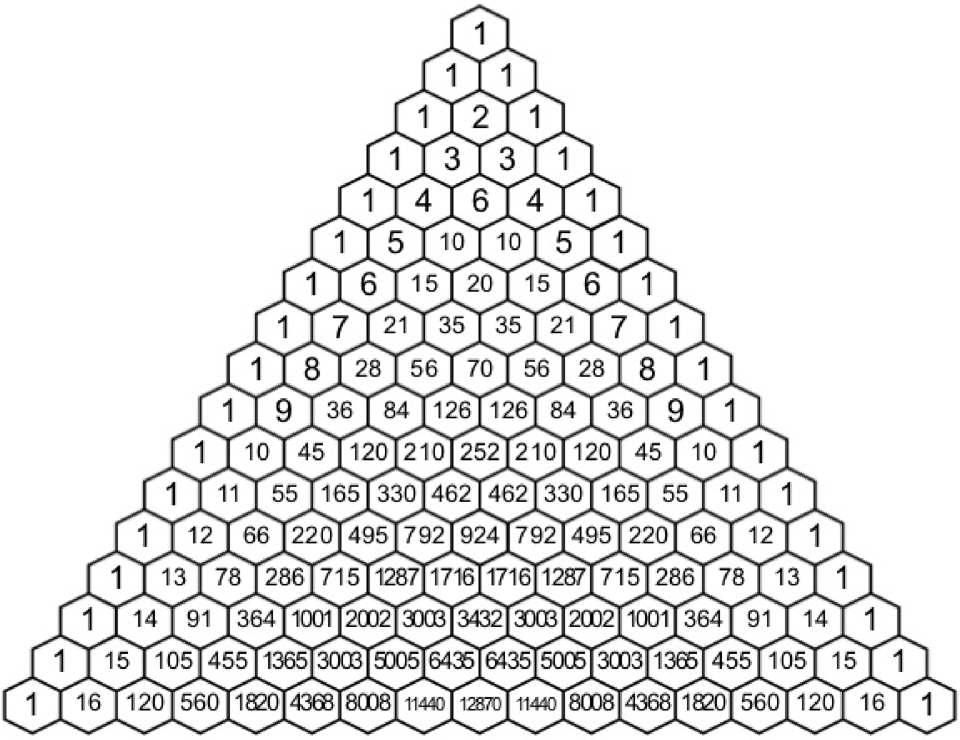
Row 0

Row 1

Row 2

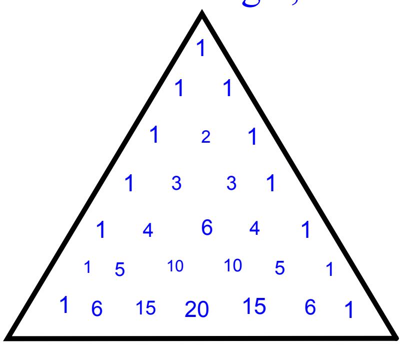
Row 3




1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1

The ***coefficients*** for a binomial expansion are found in **Pascal's Triangle**!!! The exponent on the x begins with the exponent of the binomial and progressively decreases to zero; the exponent on the y begins at zero and progresses to equal the exponent on the binomial.

So for $(x + y)^5$, the coefficients are in the 5th row of Pascal's Triangle, so the expansion is:



$$(x + y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$$
$$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

Ex.2 Use Pascal's triangle to expand the following:

a) $(x + 3)^7$

1 7 21 35 35 21 7 1

let $(a+b)^7$

$= 1a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

$= a^7 + 7a^6(3) + 21a^5(3)^2 + 35a^4(3)^3 + 35a^3(3)^4 + 21a^2(3)^5 + 7a(3)^6 + (3)^7$

$= x^7 + 21x^6 + 189x^5 + 945x^4 + 2835x^3 + 5103x^2 + 5103x + 2187$

b) $(2x - 5y)^5$ 1 5 10 10 5 1

$= (\overset{a}{\quad})^5 + 5(\overset{a}{\quad})^4(\overset{b}{\quad}) + 10(\quad)^3(\quad)^2 + 10(\quad)^2(\quad)^3 + 5(\quad)(\quad)^4 + (\quad)^5$

$= (2x)^5 + 5(2x)^4(-5y) + 10(2x)^3(-5y)^2 + 10(2x)^2(-5y)^3 + 5(2x)(-5y)^4 + (-5y)^5$

$= 32x^5 + 5(16x^4)(-5y) + 10(8x^3)(25y^2) + 10(4x^2)(-125y^3) + 5(2x)(625y^4) - 3125y^5$

$= 32x^5 - 400x^4y + 2000x^3y^2 - 5000x^2y^3 + 6250xy^4 - 3125y^5$

Ex.3 If time, show "*my*" patterning method. Ex.4 $(x - 3)^9$

$(x - 1)^8$

Use $(a+b)^8$

$= a^8b^0 + \frac{1\cdot 8}{1}a^7b^1 + \frac{8\cdot 7}{2}a^6b^2 + \frac{28\cdot 6}{3}a^5b^3 + \frac{56\cdot 5}{4}a^4b^4 + \frac{70\cdot 4}{5}a^3b^5 + \frac{56\cdot 3}{6}a^2b^6 + \frac{28\cdot 2}{7}ab^7$

$+ \frac{8\cdot 1}{8}a^0b^8$

Sub $a = x$, $b = -1$

$= (x)^8(-1)^0 + 8(x)^7(-1)^1 + 28(x)^6(-1)^2 + 56(x)^5(-1)^3 + 70(x)^4(-1)^4 + 56(x)^3(-1)^5$

$+ 28(x)^2(-1)^6 + 8(x)(-1)^7 + (x)^0(-1)^8$

$= x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$

Ex.4 $(x-3)^9$

$= 1x^9 + \frac{(1)(9)}{1}x^8(-3)^1 + \frac{(9)(8)}{2}x^7(-3)^2 + \frac{(36)(7)}{3}x^6(-3)^3$

$= x^9 + 9x^8(-3) + 36x^7(-3)^2 \ldots$

Some extra "log" stuff

$2^x = 8 \rightarrow x = \frac{\log 8}{\log 2}$

$x = 3$

$x = 3$

$2^x = 8$

$\log 2^x = \log 8$

$\frac{x \log 2}{\cancel{\log 2}} = \frac{\log 8}{\log 2}$

$x = \frac{\log 8}{\log 2}$

$\log 8$

mean $\log_{10} 8$

$3 \cdot 2^x = 24$

$\log(3 \cdot 2^x) = \log 24$

$\log 3 + \log 2^x = \log 24$

$x \log 2 = \log 24 - \log 3$

$x = \frac{\log 24 - \log 3}{\log 2}$

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 459-461 #(1 – 6)ace, 9, 11, 13 [16,18]

Today's Homework Practice includes:
p. 466 #1 – 3, (4 – 5)ace, 6, 8, 10
& Begin Review

(*p.466 #5e incorrect*?)

Attachments

PascalsTriangle.notebook