Date: ____________________________

Today's Learning Goal(s):

By the end of the class, I will be able to:

a) calculate the "future value" of an annuity earning compound interest.

Last day's work: pp. 498-499 #3 – 6, 8, 9, 11

5

p. 498 **5.** Nazir saved $900 to buy a plasma TV. He borrowed the rest at an interest rate of 18%/a compounded monthly. Two years later, he paid $1429.50 for the principal and the interest. How much did the TV originally cost?

5. $A = 1429.50$ $\quad 1429.50 = P\left(1 + \frac{0.18}{12}\right)^{24}$

$P = ?$ $\quad P = \dfrac{1429.50}{\left(1 + \frac{0.18}{12}\right)^{24}}$

$i = \frac{0.18}{12}$ $\quad \doteq \$999.998$

$n = 2 \times 12$ $\quad \doteq \$1000$

$= 24$

Original Cost $= \$1000 + \$900$

$= \$1900$

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9. Franco invests some money at 6.9%/a compounded annually and David invests some money at 6.9%/a compounded monthly. After 30 years, each investment is worth $25 000. Who made the greater original investment and by how much?

9. Franco

$A = 25\,000$, $P = ?$, $i = \frac{0.069}{1}$, $n = 30$

$25\,000 = P(1+0.069)^{30}$

$P = \frac{25\,000}{(1+0.069)^{30}} \doteq 3377.604 \doteq \$3377.60$

David

$A = 25000$, $P = ?$, $i = \frac{0.069}{12}$, $n = 30 \times 12 = 360$

$25000 = P\left(1+\frac{0.069}{12}\right)^{360}$

$P = \frac{25000}{\left(1+\frac{0.069}{12}\right)^{360}} \doteq 3173.402 \doteq \$3173.40$

Difference $= F - D = 3377.60 - 3173.40$

\$204.20 more invested by Franco.

p. 499

11. Steve wants to have $25 000 in 25 years. He can get only 3.2%/a interest compounded quarterly. His bank will guarantee the rate for either 5 years or 8 years.

- In 5 years, he will probably get 4%/a compounded quarterly for the remainder of the term.
- In 8 years, he will probably get 5%/a compounded quarterly for the remainder of the term.

a) Which guarantee should Steve choose, the 5-year one or the 8-year one?

b) How much does he need to invest?

11. a) choose the 8-yr guarantee

$A = 25\,000$, $P = ?$, $i = \frac{0.05}{4}$, $n = 17 \times 4 = 68$

$P = \frac{25\,000}{\left(1+\frac{0.05}{4}\right)^{68}} \doteq 10\,741.819 \doteq \$10\,741.82$

$P \doteq \frac{10\,741.82}{\left(1+\frac{0.032}{4}\right)^{32}} \doteq 8\,324.168 \doteq \$8\,324.17$

∴ Steve needs to invest \$8 324.17

if chose 5 year

$P_1 = \frac{25\,000}{\left(1+\frac{0.04}{4}\right)^{80}} = 11\,277.95$

$P_2 = \frac{11\,277.95}{\left(1+\frac{0.032}{4}\right)^{20}} = \$9\,616.56$

Steve would have to invest 9616.56 in the 5yr. guarantee, which is \$1292.39 more.

8.4 Annuities: Future Value

Date: Jan. 13/20

Annuity: an investment with regular deposits or withdrawals.

The **future value** of an annuity is the **sum** of all the regular payments **AND** interest earned.

Note: A **simple** annuity is an annuity in which the payments coincide with the compounding period.

An **ordinary** annuity is an annuity in which the payments are made at the end of each interval.

***Unless otherwise stated,* each annuity in this chapter is a simple, ordinary annuity.**

Ex.1 You quit smoking a pack a day "cold turkey".
You save the money for cigarettes and deposit it athe end of each half year in an account earning 6% /*a* compounded semi-annually.
Determine the future value of this annuity in 20 years.

① 1 pack a day @ \$10/pack

Each deposit = \$10 x 30 x 6

② = \$1800

$A = P(1 + i)^n$

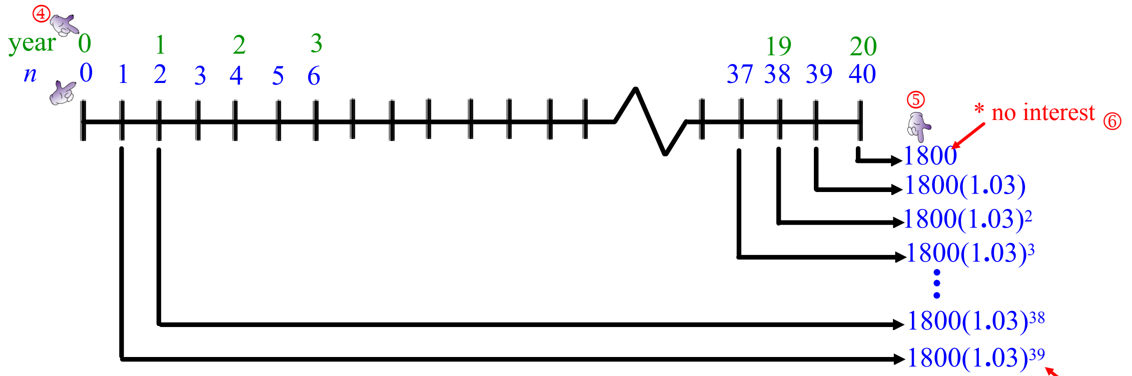
③ $i = \frac{0.06}{2}$

$= 0.03$

$n = 20 \times 2$

$= 40$

④



$S_{40} = 1800 + 1800(1.03) + 1800(1.03)^2 + ... + 1800(1.03)^{39}$

This is a **Geometric Series**, with $a = 1800$, $r = 1.03$, $n = 40$

Note: $r = 1 + i$

Use $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{40} = \frac{1800(1.03^{40} - 1)}{1.03 - 1}$

$= \$135\ 722.27$

you would have \$135 722.27 in 20 years.

Discuss Interest earned?

\$1800 x 40

= \$ 72 000

\$63 722.27

Making a formula:

Let R represent the regular payment.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{R((1+i)^n - 1)}{(1+i) - 1}$$

$$FV = \frac{R((1+i)^n - 1)}{i}$$

where R is the regular payment
i is the interest rate per compound period
n is the number of compound periods

Are there any Homework Questions you would like to see on the board?

Last day's work: pp. 498-499 #3 – 6, 8, 9, 11

Read pp. 507-508 Example 2 (both solutions)
Read the Key Ideas/Need to Know p.510

Today's Assigned Practice includes:

pp. 511-512 #2, 5ac, 6, 7

SWYK in 2 days!