1.3 Properties of Graphs of Functions

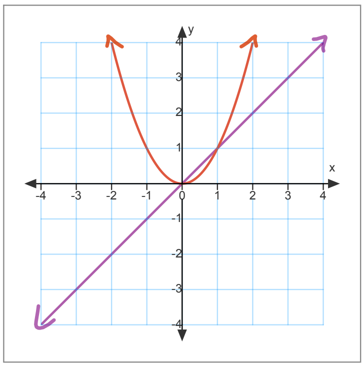
Math Learning Target:
"I can compare properties between parent functions, and within a parent function's family."

A **transformation** is a geometric operation, such as a translation, reflection and compression.

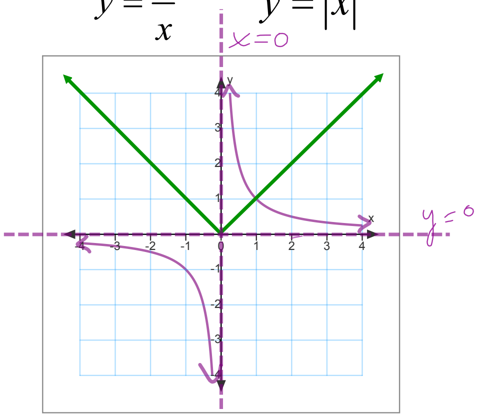
Each transformation is performed on a parent relation. There are many parent relations.
A **parent function** belongs to the set of parent relations and is the simplest function in a family of functions.
For example, the family of quadratic functions are all constructed from $y = x^2$.

Here are the *seven* parent functions that will be used often:

$y = x$ $\qquad y = x^2$



$y = \frac{1}{x}$ $\qquad y = |x|$



$y = x$ Interval(s) of increase: $(-\infty, \infty)$
Interval(s) of decrease: None
End behaviours:
as $x \to \infty, y \to \infty$
$x \to -\infty, y \to -\infty$

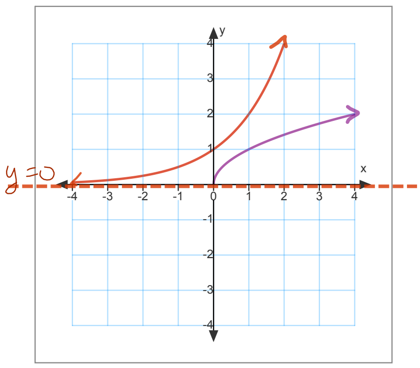
$y = \frac{1}{x}$ Interval(s) of increase: None
Interval(s) of decrease: $(-\infty, 0) \cup (0, \infty)$
End behaviours:
As $x \to \infty, y \to 0$
$x \to -\infty, y \to 0$

$y = x^2$ Interval(s) of increase: $[0, \infty)$
Interval(s) of decrease: $(-\infty, 0)$
End behaviours:
As $x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

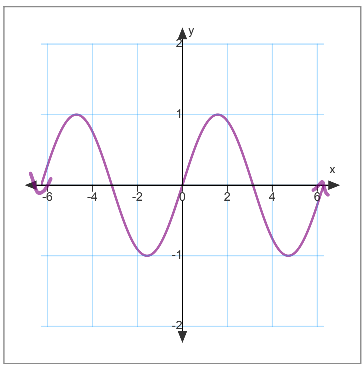
$y = |x|$ Interval(s) of increase: $[0, \infty)$
Interval(s) of decrease: $(-\infty, 0)$
End behaviours:
As $x \to \infty, y \to \infty$
$x \to -\infty, y \to \infty$

*** Recall: Did you include "0" in one interval OR the other.**

$y = \sqrt{x}$ $\qquad y = b^x$ i.e. $b = 2$, $y = 2^x$



$y = \sin(x)$



$y = \sqrt{x}$ Interval(s) of increase: $[0, \infty)$
Interval(s) of decrease: NONE
End behaviours:
As $x \to \infty, y \to \infty$

$y = 2^x$ Interval(s) of increase: $(-\infty, \infty)$
Interval(s) of decrease: NONE
End behaviours:
As $x \to \infty, y \to \infty$
$x \to -\infty, y \to 0$

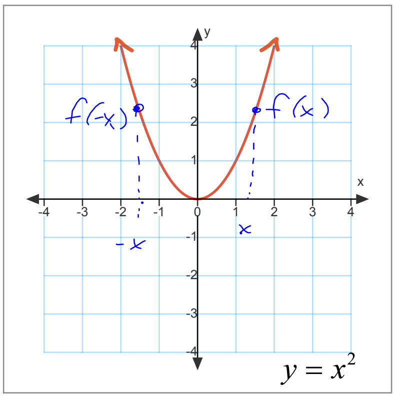
$y = \sin(x)$ End behaviours:
as $x \to \pm\infty$, y oscillates between $y = 1$ and $y = -1$

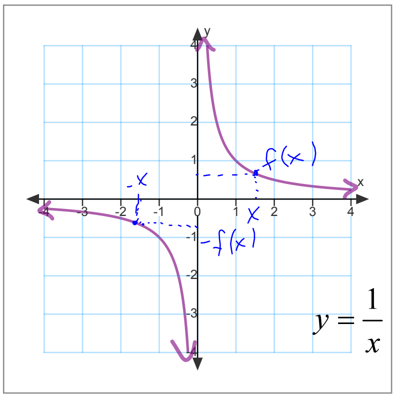
CHALLENGE! Can you determine expressions for the intervals of increase and intervals of decrease?

1.3 Properties of Graphs of Functions (Spring 2020)-s20 — February 6, 2020

"Symmetry"

A function is **odd** when $f(-x) = -f(x)$ — A function is **even** when $f(-x) = f(x)$





Graphically, a function is even when

it is symmetric about the y-axis (only)

Graphically, a function is odd when

→ it has rotational symmetry about the origin.

→ it is symmetric about both axes.

→ double reflection → reflecting in the x-axis, then y-axis

Ex.1 Is $y = x^2$ even, odd, or neither? Prove algebraically.

$f(x) = x^2$

Consider $f(-x)$

$= (-x)^2$

$= (-1)^2(x)^2$

$= 1x^2$

$= x^2$

$\because f(-x) = f(x)$

$\therefore f(x)$ is even

} conclusion NEEDED.

Ex.2 Is $y = \frac{1}{x}$ even, odd, or neither? Prove algebraically.

$f(x) = \frac{1}{x}$

Consider $f(-x) = \frac{1}{(-x)}$

$= -\frac{1}{x}$

$= -f(x)$

→ $\because f(-x) = -f(x)$

$\therefore f(x)$ is odd

Do: pg. 23 #3*, 4ad, 5**, 6, 7, 8, 10***, 15

* Error in answer: the function can be derived from any $y=b^x$, for any valid "b"),

** The instructions are poor. Simply apply what was learned today in the lesson.

***In #10a, in the instructions for the question change $(-\infty, -2)$ to $(-\infty, 2]$

positive 2

square bracket

YES, you have permission to write in the textbook to make this change!