Questions from last day's homework?

Read and understand p. 75 "**Need to Know**".
Complete: pp. 76-78 #2, 6, 7, 10, 12, 13

p. 77 **7.** Shelly has a cell phone plan that costs \$39 per month and allows her 250 free anytime minutes. Any minutes she uses over the 250 free minutes cost \$0.10 per minute. The function

$$C(m) = \begin{cases} 39, \text{ if } 0 \le m \le 250 \\ 0.10(m - 250) + 39, \text{ if } m > 250 \end{cases}$$

can be used to determine her monthly cell phone bill, where $C(m)$ is her monthly cost in dollars and m is the number of minutes she talks. Discuss how the average rate of change in her monthly cost changes as the minutes she talks increases.

$200 \le m \le 249$

$$\text{aroc} = m_{secant} = \frac{\Delta C}{\Delta t} = \frac{C(249) - C(200)}{249 - 200} = \frac{39 - 39}{49} = 0$$

∴ on average, the rate of change in price is 0 from 0 to 250 minutes

$255 \le m \le 265$

$$\frac{\Delta C}{\Delta t} = \frac{C(265) - C(255)}{265 - 255}$$

$$= \frac{0.1(265-250)+39-(0.1(255-250)+39)}{10}$$

$$= \frac{0.1(15)+39-(0.1(5)+39)}{10}$$

$$= \frac{1.5+39-(0.5+39)}{10}$$

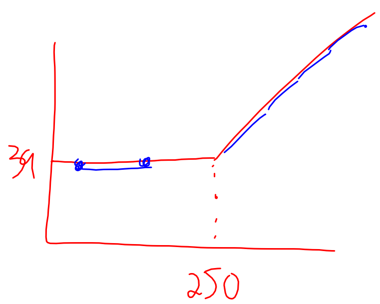
$$= \frac{40.50 - 39.50}{10}$$

$$= \frac{1}{10}$$

$$= 0.1$$

$300 \le m \le 320$

$$\text{aroc} = \frac{C(320) - C(300)}{320 - 300} = \frac{46 - 44}{20} = \frac{2}{20} = 0.10$$



p. 76

6. What is the average rate of change in the values of the function $f(x) = 4x$ from $x = 2$ to $x = 6$? What about from $x = 2$ to $x = 26$? What do your results indicate about $f(x)$?

6. The function is $f(x) = 4x$. To find the average rate of change find $\frac{\Delta f(x)}{\Delta x}$. The rate of change from $x = 2$ to $x = 6$ is:

$$\frac{f(6) - f(2)}{6 - 2} = \frac{4(6) - 4(2)}{6 - 2} = \frac{24 - 8}{6 - 2} = \frac{16}{4} = 4$$

The rate of change from 2 to 26 is:

$$\frac{f(26) - f(2)}{26 - 2} = \frac{4(26) - 4(2)}{26 - 2} = \frac{104 - 8}{26 - 2} = \frac{96}{24} = 4$$

The average rate of change is always 4 because the function is linear, with a slope of 4.

There is something called the "constant slope property" which means that on a linear function, you will always get the same slope, in this case, 4.

$y = mx + b$

$y = 4x + 0$

$m = 4$

p. 77 **10.** A company that sells sweatshirts finds that the profit can be modelled by $P(s) = -0.30s^2 + 3.5s + 11.15$, where $P(s)$ is the profit, in thousands of dollars, and s is the number of sweatshirts sold (expressed in thousands).

a) Calculate the average rate of change in profit for the following intervals.

i) $1 \le s \le 2$ ii) $2 \le s \le 3$ iii) $3 \le s \le 4$ iv) $4 \le s \le 5$

b) As the number of sweatshirts sold increases, what do you notice about the average rate of change in profit on each sweatshirt? What does this mean?

c) Predict if the rate of change in profit will stay positive. Explain what this means.

$$\frac{\Delta P}{\Delta n} = \frac{P(\quad) - P(\quad)}{\quad - \quad}$$

$-0.30s^2 + 3.5s + 11.15$

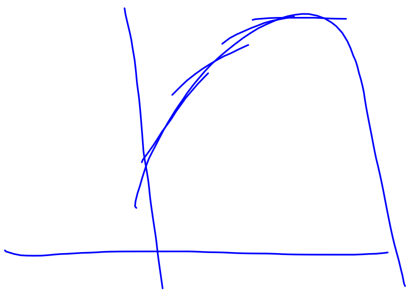
$$\frac{\Delta P}{\Delta n} = \frac{P(2) - P(1)}{2 - 1}$$

$$= \frac{-0.3(2)^2 + 3.5(2) + 11.15 - \left(-0.3(1)^2 + 3.5(1) + 11.15\right)}{1}$$

$$= \frac{16.95 - 14.35}{1}$$

$= 2.60 \ \frac{\text{thousand}}{\text{thousand}} \rightarrow \frac{\$2600}{1000}$

$= \$2.60/\text{shirt}$



p. 78 **12.** Your classmate was absent today and phones you for help with today's lesson. Share with your classmate

C

a) two real-life examples of when someone might calculate an average rate of change (one positive and one negative)
b) an explanation of when an average rate of change might be useful
c) an explanation of how an average rate of change is calculated

12. Answers may vary. For example:
a) Someone might calculate the average increase in the price of gasoline over time. One might calculate the average decrease in the price of computers over time.
b) An average rate of change would be useful when there are several different rates of change over a specific interval.
c) The average rate of change is found by taking the change in y for the specified interval and dividing it by the change in x over that same interval.

p. 78

13. Vehicles lose value over time. A car is purchased for \$23 500, but is worth only \$8750 after eight years. What is the average annual rate of change in the value of the car, as a percent?

$$ARC = \frac{23500 - 8750}{0 - 8}$$

$$= \frac{14750}{-8}$$

$$= -\$1843.75/yr.$$

$$\% \text{ Change} = \frac{\text{change/diff.}}{\text{original}} \times 100\%$$

$$\frac{P(8)-P(0)}{8-0}$$

$$= \frac{8750-23500}{8}$$

$$= \frac{-14750}{8}$$

$$= \frac{-1843.75}{23500} \times 100$$

$$\doteq -7.84$$

$$\doteq -7.8\%$$

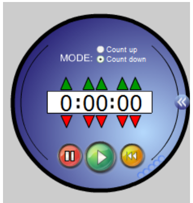
2.2 Estimating Instantaneous Rates of Change from Tables of Values and Equations (Part 1)

Math Learning Target:

"I understand what average rate of change and instantaneous rate of change are, and the difference between the them.
Also, I can determine a reasonable estimate for the instantaneous rate of change using the methods presented, and interpret the result."

***INVESTIGATE** the Math (Page 79, A - G)*

10 min. timer



Time, t (s)	6.0	6.2	6.4	6.6	6.8	7.0
Distance, $d(t)$ (cm)	208.39	221.76	235.41	249.31	263.46	277.84

A. Calculate the average rate of change in the distance that thepebble fell during each of the following time intervals.

1) $6.0 \leq t \leq 6.4$ **2)** $6.2 \leq t \leq 6.4$ **3)** $6.4 \leq t \leq 7.0$

preceding interval

following interval

$$aroc = \frac{\Delta d}{\Delta t}$$
$$= \frac{235.41 - 208.39}{6.4 - 6.0}$$
$$= \frac{27.02}{0.4}$$
$$= 67.55 \text{ cm/s}$$

$\doteq 68.25$ cm/s

$\doteq 70.716$ cm/s

4) $6.4 \leq t \leq 6.8$ **5)** $6.4 \leq t \leq 6.6$

$\doteq 70.125$ cm/s $= 69.5$ cm/s

B The ***instantaneous rate of change*** in the distance at t = 6.4 s is about 69 cm/s.
Look at smallest intervals using 6.4 as an endpoint.

C $6.2 \leq t \leq 6.6$
$= 68.875$ m/s

centred interval

D It is the place where we want to know the instantaneous rate of change.

E By seeing the rate of change on both sides of 6.4, it will be easier to guess the rate of change at 6.4 because it will need to be somewhere in between those calculations. The best estimates come from the smallest intervals on either side of 6.4 s.

F 6.4 s is the midpoint of this interval. It balances the estimation error that results when only a single interval is used on either side of the value (above or below) for which the instantaneous rate of change is to be determined.

G No. I am able to guess what it might be but there are no values to the right of t = 7.0 to verify that the rate of change chosen might be correct.

We now understand the concept of the average rate of change.

The ***instantaneous rate of change*** is the rate of change at one specific value ***x = a*** for a function ***y =f(x)***.
We begin by learning how to *approximate* it.

preceding interval
an interval of the independent variable of the form $a - h \le x \le a$, where h is a small positive value; used to determine an average rate of change

following interval
an interval of the independent variable of the form $a \le x \le a + h$, where h is a small positive value; used to determine an average rate of change

HW pp. 85-88 #1, 2a, 3, 8, 14

Ex.1:
A bacteria culture is growing exponentially and the population, P, is given by $P(n) = 200(1.2)^n$, where n is the number of hours. *Estimate,* to the nearest tenth, the instantaneous rate of change at 5 hours, by using:
a) at least three preceding intervals
b) at least three following intervals

$iroc = \frac{\Delta P}{\Delta n}$ $4.5 \le n \le 5$

$\frac{\Delta P}{\Delta n} = \frac{P(\quad) - P(\quad)}{\quad - \quad}$

$\frac{\Delta P}{\Delta n} = \frac{200(1.2)^{5} - 200(1.2)^{4.5}}{5 - 4.5}$

$= 86.72$ bacteria/hour

$4.9 \le n \le 5$

$\frac{\Delta P}{\Delta n} = \frac{P(5) - P(4.9)}{5 - 4.9}$

$\frac{\Delta P}{\Delta n} = \frac{200(1.2)^{5} - 200(1.2)^{4.9}}{5 - 4.9}$

$\doteq \frac{497.664 - 488.672}{0.1}$

$\doteq 89.91$ bacteria/h

$4.99 \le n \le 5$

$\frac{\Delta P}{\Delta n} \doteq 90.65$

$\doteq 90.7$

$4.999 \le n \le 5$

$\frac{\Delta P}{\Delta n} \doteq 90.72$

$\doteq 90.7$

$\frac{\Delta P}{\Delta n} = \frac{P(\quad) - P(\quad)}{\quad - \quad}$

$5.0 \le n \le 5.1$ $\frac{\Delta P}{\Delta n} =$

$\frac{\Delta P}{\Delta n} = \frac{200(1.2)^{5.1} - 200(1.2)^{5}}{5.1 - 5}$

$\doteq 91.56$ bacteria/hour

$\doteq 91.6$

$5.0 \le n \le 5.01$

$\frac{\Delta P}{\Delta n} \doteq 90.81$

$\doteq 90.8$

$5.0 \le n \le 5.001$

$\frac{\Delta P}{\Delta n} \doteq 90.74$

$\doteq 90.7$

Estimate $= \frac{90.72 + 90.74}{2}$

$\doteq 90.73$

it appears the populaton is growing
at exactly 5 hours by about 90.7 bacteria per hour.