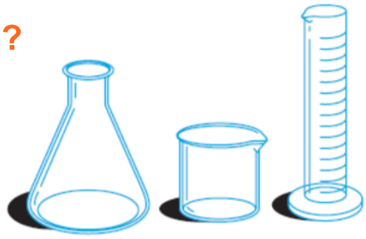
2.4 Using Rates of Change to Create a Graphical Model

Math Learning Target:

"I can represent written and verbal descriptions of rates of change using graphs.
I can interpret rates of change from graphs that I am given."

Use Vertical White Boards?

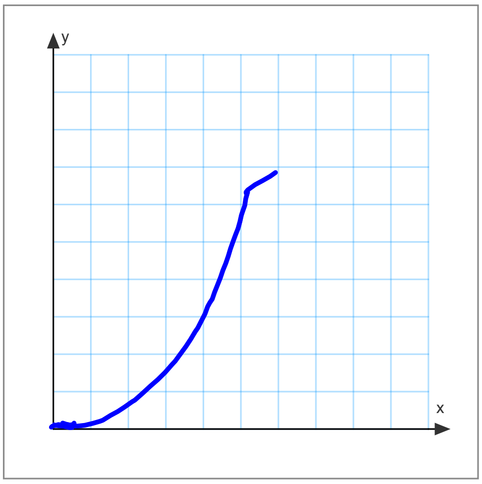
Ex.1 (p.97 Ex.2)



A flask, a beaker, and a graduated cylinder are being filled with water. The rate at which the water flows from a hose is the same when filling all three containers. Draw possible water level versus time sketches for the three containers.

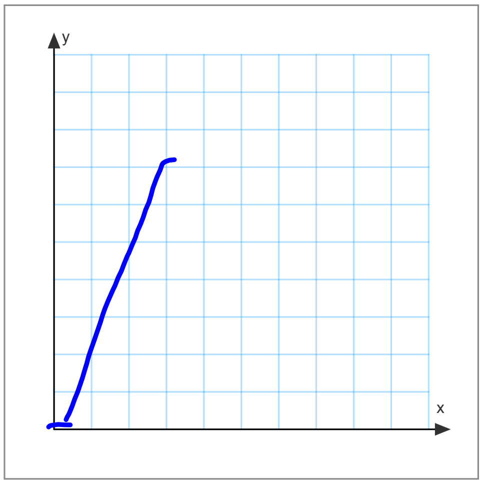
Assume the time interval begins the moment the water begins to pour from the hose, and the hose stays immediately above the current water level at all times.

height (cm)

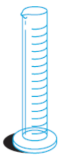


time (sec)

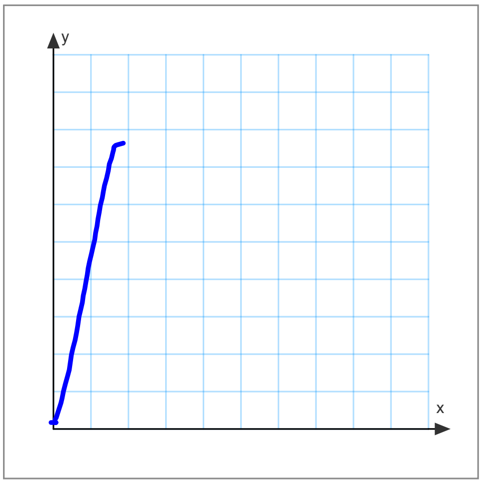
height (cm)



time (sec)



height (cm)



time (sec)

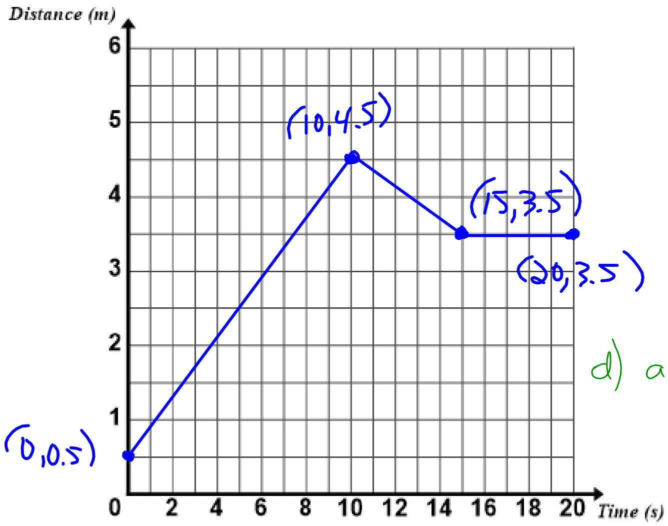
Ex.2 **Use Vertical White Boards?** (p.100 Ex.4)

...an additional... ...at a constant rate...

Adam and his friend are testing a motion sensor. Adam stands 0.5 m in front of the sensor and then walks 4 m away from it at a constant rate for 10 s. Next, Adam walks 1 m toward the sensor for 5 s and then waits there for another 5 s.

a) Draw a distance versus time graph for Adam's motion sensor walk.
b) What is the average rate of change in his distance in the first 10 s?
c) What are the instantaneous rates of change at $t = 1$ s and $t = 7$ s?
d) What is the average rate of change in the next 5 s?
e) What are the instantaneous rates of change at $t = 12$ s and $t = 14$ s?
f) What is the instantaneous rate of change at $t = 18$ s?
g) Draw a speed versus time graph for Adam's motion sensor walk.

...average...



b) $aroc = m_{secant}$

$= \frac{\Delta d}{\Delta t}$

$= \frac{4.5 - 0.5}{10 - 0}$

$= \frac{4}{10}$

$= \frac{2}{5}$

$= 0.4$ m/s

c) $iroc = m_{tangent}$

but at $t = 1$ and $t = 7$

$iroc = 0.4$ m/s

Same as m_{secant} since the graph is linear $[0, 10]$

d) $aroc = m_{secant}$

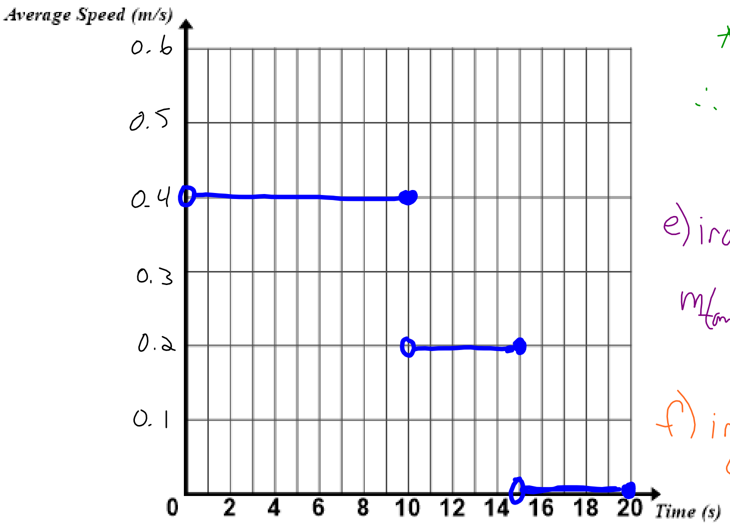
$= \frac{3.5 - 4.5}{15 - 10}$

$= -\frac{1}{5}$

$= -0.2$ m/s

* speed is NON-NEGATIVE

∴ moving TOWARDS the sensor at 0.2 m/s



e) iroc at $t = 12$ + $t = 14$

$m_{tangent} = -0.2$

$= m_{secant}$

∵ linear $(10, 15]$

f) iroc at $t = 18$

$\hookrightarrow = \frac{0}{1}$

$= 0$ m/s

∵ horizontal $(15, 20]$

Follow the instructions provided in our Google Classroom to explore the following applet s using GeoGebra.

https://www.geogebra.org/m/ImfrMzGo https://www.geogebra.org/m/astu2rT4

ENTERTAINMENT: pp.103-106 #1, 2*, 3 to 9*, 10, 11, 14
*** in #2, the answer in the back has a small error. Do you know what it is?**
Also, the answer for #9 in the back has some mistakes.

10 4a

4. For the function $f(x) = 6x^2 - 4$, estimate the instantaneous rate of change for the given values of x.

a) $x = -2$ **b)** $x = 0$ **c)** $x = 4$ **d)** $x = 8$

$-2.001 \le x \le -1.999$

$$aroc = \frac{f(-1.999) - f(-2.001)}{-1.999 - (-2.001)}$$

$$= \frac{6(-1.999)^2 - 4 - [6(-2.001)^2 - 4]}{0.002}$$

$$= \frac{19.976006 - 20.024006}{0.002}$$

$$= \frac{-0.048}{0.002}$$

$$= -24$$

10. To make a snow person, snow is being rolled into the shape of a sphere. The volume of a sphere is given by the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius in centimetres. Use two different methods to estimate the instantaneous rate of change in the volume of the snowball with respect to the radius when $r = 5$ cm.

$iroc = m_{tangent}$

$$= \frac{V(5+h) - V(5)}{5+h-5}, \text{ as } h \to 0$$

$$= \frac{\frac{4}{3}\pi(5+h)^3 - \frac{4}{3}\pi(5)^3}{h}, \text{ as } h \to 0$$

$$= \frac{\frac{4}{3}\pi(5^3 + 3(5)^2h + 3(5)h^2 + h^3) - \frac{4}{3}\pi(125)}{h}, \text{ as } h \to 0$$

$$= \frac{\frac{4}{3}\pi[125 + 75h + 15h^2 + h^3) - \frac{4}{3}\pi(125)}{h}, \text{ as } h \to 0$$

$$= \frac{\frac{4}{3}\pi(125) + \frac{4}{3}\pi[75h + 15h^2 + h^3] - \frac{4}{3}\pi(125)}{h}, \text{ as } h \to 0$$

$$= \frac{\frac{4}{3}\pi \cancel{h}[75 + 15h + h^2]}{\cancel{h}}, \text{ as } h \to 0$$

$$= \frac{4}{3}\pi(75 + 15h + h^2), \text{ as } h \to 0$$

$$= \frac{4}{3}\pi(75 + 0 + 0)$$

$$= \frac{4}{\cancel{3}}\pi(\cancel{75}^{25})$$

$$= 100\pi$$