Summary from Chapter 3 Topic 2: *Characteristics of Polynomial Functions*

In Summary

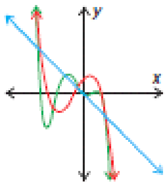
Key Ideas

- Polynomial functions of the same degree have similar characteristics.
- The degree and the leading coefficient in the equation of a polynomial function indicate the end behaviours of the graph.
- The degree of a polynomial function provides information about the shape, turning points, and zeros of the graph.

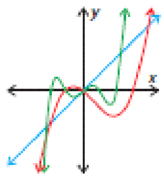
Need to Know

End Behaviours

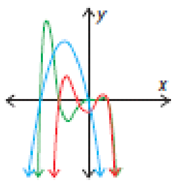
- An odd-degree polynomial function has opposite end behaviours.
  - If the leading coefficient is negative, then the function extends from the second quadrant to the fourth quadrant; that is, as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$.



  - If the leading coefficient is positive, then the function extends from the third quadrant to the first quadrant; that is, as $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$.



- An even-degree polynomial function has the same end behaviours.
  - If the leading coefficient is negative, then the function extends from the third quadrant to the fourth quadrant; that is, as $x \to \pm\infty$, $y \to -\infty$.



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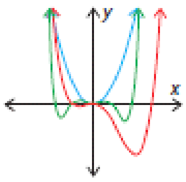


Turning Points

- A polynomial function of degree n has at most $n - 1$ turning points.

Number of Zeros

- A polynomial function of degree n may have up to n distinct zeros.
- A polynomial function of odd degree must have at least one zero.
- A polynomial function of even degree may have no zeros.

Symmetry

- Some polynomial functions are symmetrical in the y-axis. These are even functions, where $f(-x) = f(x)$.
- Some polynomial functions have rotational symmetry about the origin. These are odd functions, where $f(-x) = -f(x)$.
- Most polynomial functions have no symmetrical properties. These are functions that are neither even nor odd, with no relationship between $f(-x)$ and $f(x)$.

3.3 Characteristics of Polynomial Functions (in *Factored Form)*

Math Learning Target:

"I can identify properties of polynomial functions when expressed in factored form.
I can express any polynomial function in its factored form, and then graph it."

Recall: To **factor** a number (or expression) means to determine the numbers (or expressions) that divide into it with a remainder of zero.

Recall: A **prime number** is a positive number that has only two unique factors: 1 and itself. Note that the number 1 is not prime.

24
$= 3 \times 8$
$= 3 \times 2 \times 2 \times 2$

or $24 = (8)(3) + 0$ remainder

$x^3 - 13x^2 - 30x$
$= x(x^2 - 13x - 30)$
$= x(x-15)(x+2)$

$\ne x^2(x-13) - 30x$
NOT considered factored, because it is NOT a PRODUCT.

Recall: The **zeros** of a function $y=f(x)$ are all real numbers x such that $f(x) = 0$. They correspond to the x-intercepts of the function $y=f(x)$. In the *INVESTIGATE* from a previous class, you learned that a polynomial function of degree n may have up to n distinct zeros.

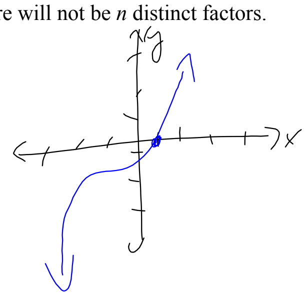
$f(x) = x^2 - 6x + 8$ $= (x-4)(x-2)$ if $f(x) = 0$ $0 = (x-4)(x-2)$ $x = 4$ or $x = 2$ $y = (x-3)^2 - 1$ $(3,-1)$ $\therefore$ 2 distinct zeros	$f(x) = x^2 - 6x + 9$ $= (x-3)(x-3)$ $= (x-3)^2$ if $f(x) = 0$ $0 = (x-3)^2$ $\therefore x = 3$ (Order 2) $(3,0)$ $\therefore$ 1 distinct zero $y = (x-3)^2$	$f(x) = x^2 - 6x + 10$ $\times$ DNF: 2 possibilities $\to x \in \mathbb{Q}$, no roots $= x^2 - 6x + 9 - 9 + 10$ $= (x-3)^2 + 1$ $v(3,1)$ above the axis, opens up below the x-axis, opens down

If a polynomial function $y=f(x)$ with degree n has exactly n distinct zeros, then the factored forms are:

degree = 1	linear	$f(x) = a(x-p)$
degree = 2	quadratic	$f(x) = a(x-p)(x-q)$
degree = 3	cubic	$f(x) = a(x-p)(x-q)(x-r)$
degree = 4	quartic	$f(x) = a(x-p)(x-q)(x-r)(x-s)$
degree = 5	quintic	$f(x) = a(x-p)(x-q)(x-r)(x-s)(x-t)$
etc...	etc...	etc...

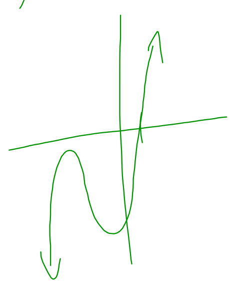
If a polynomial function $y=f(x)$ with degree n has less than n distinct zeros, but at least one zero, the function can still be expressed in factored form, but there will not be n distinct factors.

ex) $f(x) = (x-1)(x^2+x+1)$



or

$f(x) = (x-1)(x^2+5x+7)$



Ex.1 Sketch $f(x)=2(x+1)^2(x-3)$

$y=2(x+1)^2(x-3)$

lead coefficient : +2 } + degree 3

end behaviours :

as $x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

Zeros: $-1, 3$

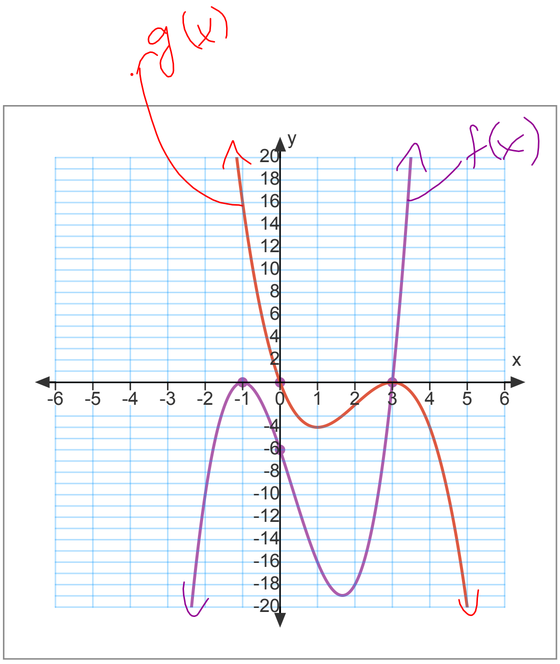
order: $2, 1$

y-int, let $x=0$: $f(0)$

$=2(0+1)^2(0-3)$

$=2(1)^2(-3)$

$=-6$



Ex.2 Sketch $g(x)=-x^3+6x^2-9x$

$=-x(x^2-6x+9)$

$=-x(x-3)^2$

y-int: $f(0)=0$

Zeros: $0, 3$

Order: $1, 2$

$y=-x^3+6x^2-9x$

Ex. 3 a) Determine the equation of the quartic function with zeros -2, ¾, 5(order 2) and a y-intercept of y= -37.5. $\to (0, -37.5)$

b) Determine at least two other functions that belong to the same family.

a) $y=a(x+2)(x-\frac{3}{4})(x-5)^2$

$-37.5=a(0+2)(0-\frac{3}{4})(0-5)^2$

$-37.5=a(2)(-\frac{3}{4})(25)$

$\vdots$

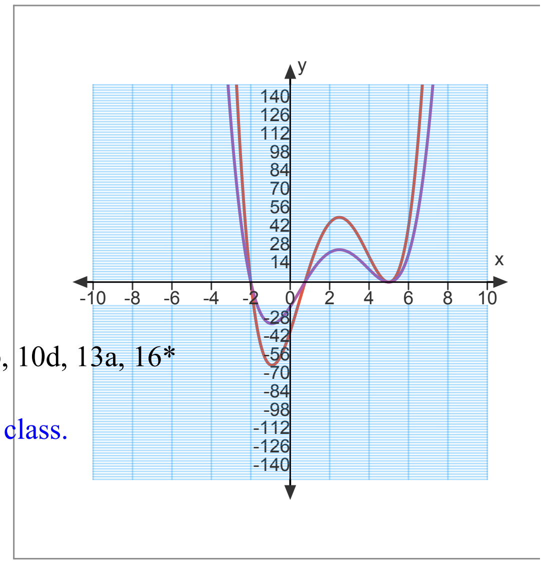
$a=1$

$\therefore y=1(x+2)(x-\frac{3}{4})(x-5)^2$

$y=(x+2)(x-0.75)(x-5)^2$

$y=2(x+2)(x-0.75)(x-5)^2$

$y=0.5(x+2)(x-0.75)(x-5)^2$



Now complete pp.146-148 #1, 2a, 4b, 6be, 8ab, 9ab, 10d, 13a, 16*

* for 16b you will need to use desmos

A formative assessment of Topics 1, 2 and 3 is next class.

A Summary of main points from today's lesson....

In Summary

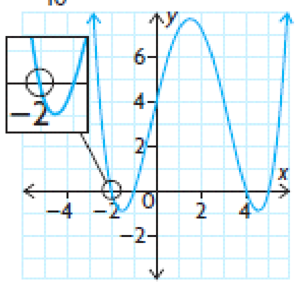
Key Idea

- The zeros of the polynomial function $y = f(x)$ are the same as the roots of the related polynomial equation, $f(x) = 0$.

Need to Know

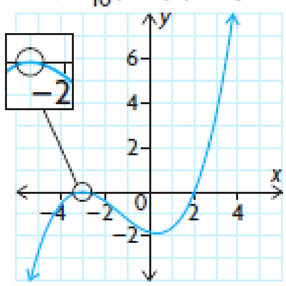
- To determine the equation of a polynomial function in factored form, follow these steps:
  - Substitute the zeros $(x_1, x_2, \ldots, x_n)$ into the general equation of the appropriate family of polynomial functions of the form $y = a(x - x_1)(x - x_2)\ldots(x - x_n)$.
  - Substitute the coordinates of an additional point for x and y, and solve for a to determine the equation.
- If any of the factors of a polynomial function are linear, then the corresponding x-intercept is a point where the curve passes through the x-axis. The graph has a linear shape near this x-intercept.

$y = \frac{1}{10}(x + 2)(x + 1)(x - 4)(x - 5)$



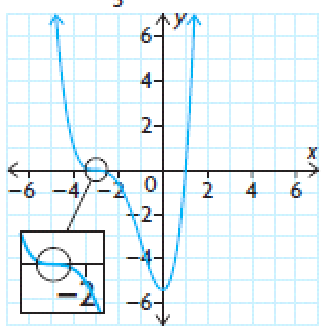
- If any of the factors of a polynomial function are squared, then the corresponding x-intercepts are turning points of the curve and the x-axis is tangent to the curve at these points. The graph has a parabolic shape near these x-intercepts.

$y = \frac{1}{10}(x + 3)^2(x - 2)$



- If any of the factors of a polynomial function are cubed, then the corresponding x-intercepts are points where the x-axis is tangent to the curve and also passes through the x-axis. The graph has a cubic shape near these x-intercepts.

$y = \frac{1}{5}(x + 3)^3(x - 1)$



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