3.6 Factoring Polynomials: *Part 1*

Math Learning Target:
"I can state the Remainder Theorem and the Factor Theorem.
By the end of next class, I can always apply it, when it is applicable."

Given: $p(x) = x^3 + 2x^2 - 5x - 6$

Evaluate: $p(-2)$

$p(-2) = (-2)^3 + 2(-2)^2 - 5(-2) - 6$
$= -8 + 2(4) + 10 - 6$
$= -8 + 8 + 4$
$= 4$

$p(1) = (1)^3 + 2(1)^2 - 5(1) - 6$
$= 1 + 2 - 5 - 6$
$= -8$

Divide $p(x)$ by:

a) $x + 2$

$$
\begin{array}{r|l}
 & x^2 \quad\quad - 5 \\
x+2 & x^3 + 2x^2 - 5x - 6 \\
 & -(x^3 + 2x^2) \\
 & \hline 0 + 0 - 5x - 6 \\
 & -(-5x - 10) \\
 & \hline 4 \ R
\end{array}
$$

b) $x - 1$

$$
\begin{array}{r|rrrr}
1 & 1 & 2 & -5 & -6 \\
 & \downarrow & 1 & 3 & -2 \\
\hline
 & 1 & 3 & -2 & -8 \\
\end{array}
$$

R

What do you notice?

The remainder can be found easily,
without using long (or synthetic) division.

The Remainder Theorem

- Given polynomial function $p(x)$,
  If $p(x)$ is divided by $x - a$, where $a \in \mathbb{R}$, the remainder is $p(a)$.

For the same polynomial function $p(x) = x^3 + 2x^2 - 5x - 6$

a) Evaluate $p(2)$

$p(2) = (2)^3 + 2(2)^2 - 5(2) - 6$

$= 8 + 8 - 10 - 6$

$= 0$

b) Divide by $x - 2$

$$\begin{array}{c|cccc} 2 & 1 & 2 & -5 & -6 \\ & \downarrow & 2 & 8 & 6 \\ \hline & 1 & 4 & 3 & 0R \end{array}$$

$\boxed{\text{as expected}}$

- Thus, when $p(x)$ is divided by x-2, the remainder is zero.

$\therefore x^3 + 2x^2 - 5x - 6 = (x-2)(x^2 + 4x + 3) + 0$

The Factor Theorem

iff

- **Given polynomial function $f(x)$,**
  #1) If $f(a) = 0$, then $x - a$ is a factor of $f(x)$, and,
  #2) If $x - a$ is a factor, then $f(a) = 0$, for all a $\mathbb{R}$

Factor this polynomial completely, if it is factorable: $f(x) = x^3 - 6x^2 - x + 30$

[try $\pm1, \pm2, \pm3, \pm5, \pm6$
$\pm30, \pm15, \pm10$]

$f(1) = 1 - 6 - 1 + 30 = 24$

$f(-1) = -1 - 6 + 1 + 30 = 24$

$f(2) = 8 - 6(4) - 2 + 30 = 12$

$f(-2) = -8 - 6(4) + 2 + 30 = 0$

$\therefore x+2$ is a factor of $f(x)$

-2	1	-6	-1	30
	$\downarrow$	-2	16	-30
	1	-8	15	0 R

0 R as expected

$f(x) = x^3 - 6x^2 - x + 30$

$= (x+2)(x^2 - 8x + 15)$

or

$= (x+2)\,g(x)$

if $g(x) = x^2 - 8x + 15$

$g(5) = 0$

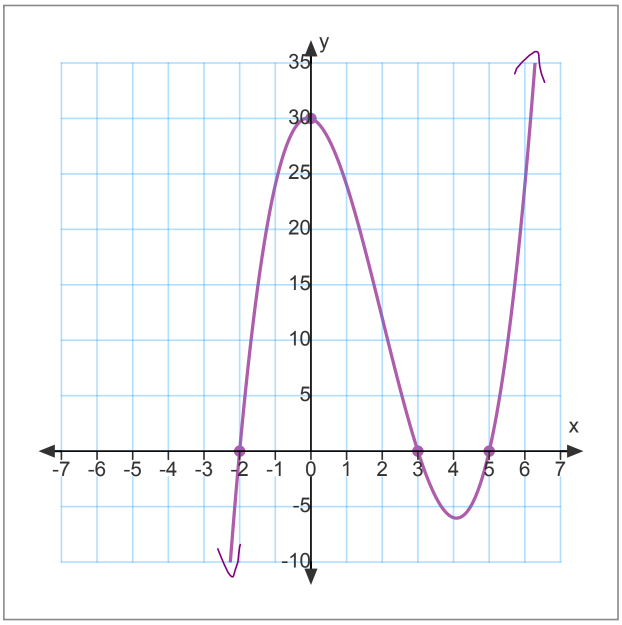
5	1	-8	15
	$\downarrow$	5	-15
	1	-3	0 R

$\therefore g(x) = (x-5)(x-3)$

$0 = (x+2)(x-5)(x-3)$

zeros: $-2, 5, 3$
order: 1, 1, 1

y-int, let $x = 0$
$y = 30$



$y = x^3 - 6x^2 - x + 30$

Today's Assigned Practice
pp. 176-177 #1 to 3, 4ace, 5ace

Last day's Work

p. 122 #1d, 2

pp. 184-185 #1, 2*, 3, 4d, 5d, 6, 8bcd, 9cef, 10ad, 12cd

**the answer is wrong in the back for #2*

p. 184

5. For each of the following, write the equations of three quartic functions that have the given zeros and belong to the same family of functions.

a) $-6, 2, 5, 8$ **c)** $0, -1, 9, 10$

b) $4, -8, 1, 2$ **d)** $-3, 3, -6, 6$

$y = a(x+3)(x-3)(x+6)(x-6)$

$y = 7(x+3)(x-3)(x+6)(x-6)$

10. Calculate each of the following using long division.

d) $(x^5 - 8x^3 - 7x - 6) \div (x^4 + 4x^3 + 4x^2 - x - 3)$

$x^5 - 8x^3 - 7x - 6$

$$\begin{array}{r} x - 4 \\ x^4 + 4x^3 + 4x^2 - x - 3 \,\big)\, \overline{x^5 + 0x^4 - 8x^3 + 0x^2 - 7x - 6} \\ -\left(x^5 + 4x^4 + 4x^3 - x^2 - 3x\right) \\ \hline -4x^4 - 12x^3 + x^2 - 4x - 6 \\ -\left(-4x^4 - 16x^3 - 16x^2 + 4x + 12\right) \\ \hline 4x^3 + 17x^2 - 8x - 18 \end{array}$$

Division Statement:

dividend = divisor x quotient + remainder

$\therefore x^5 - 8x^3 - 7x - 6 = (x-4)(x^4 + 4x^3 + 4x^2 - x - 3) + 4x^3 + 17x^2 - 8x - 18$