From yesterday's entertainment:
p.177 #6de, 7e, 8e, 9, 10, 14, 16, 17

p.177

6. Factor each polynomial using the factor theorem.

e) $x^5 - 5x^4 - 7x^3 + 29x^2 + 30x$ d) $4x^4 + 7x^3 - 80x^2 - 21x + 270$

$f(x) = x^5 - 5x^4 - 7x^3 + 29x^2 + 30x$

$= x(x^4 - 5x^3 - 7x^2 + 29x + 30)$

$f(-1) = 1 + 5 - 7 - 29 + 30$

$\because f(-1) = 0 \therefore x+1$ is a factor

-1	1	-5	-7	29	30
	↓	-1	6	1	-30
	1	-6	-1	30	0R

$g(x) = x^3 - 6x^2 - 1x + 30$

$\therefore f(x) = (x+1)g(x)$

$= (x+1)(x-5)(x^2 - x - 6)$

$= (x+1)(x-5)(x-3)(x+2)$

$g(5) = 125 - 6(25) - 5 + 30$

$= 125 - 150 - 5 + 30$

$= 0$

$\because g(5) = 0 \therefore x-5$ is a factor

5	1	-6	-1	30
	↓	5	-5	-30
	1	-1	-6	0R

From yesterday's entertainment:
p.177 #6de, 7e, 8e, 9, 10, 14, 16, 17

p.177

7. Factor fully. 8. Use the factored form of f(x) to sketch the graph of each function.

e) $f(x) = x^3 - x^2 + x - 1$

7e) $= x^2(x-1) + 1(x-1)$

$0 = (x-1)(x^2+1)$

$x = 1$ or $x^2 + 1 = 0$

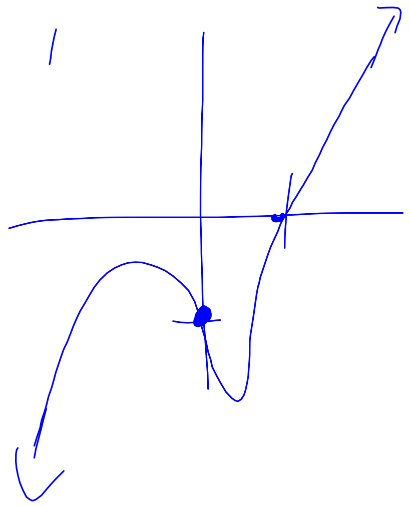
$x^2 = -1$

No Real Roots

8e) Zeros: 1

order: 1

$f(0) = -1$ ← y-int



9. The polynomial $12x^3 + kx^2 - x - 6$ has $2x - 1$ as one of its factors. Determine the value of k.

$2x - 1 = 0$

if $f(x) = 12x^3 + kx^2 - x - 6$ $\quad x = \frac{1}{2}$ $\quad \therefore f(\frac{1}{2}) = 0$

$f(\frac{1}{2}) = 12(\frac{1}{2})^3 + k(\frac{1}{2})^2 - (\frac{1}{2}) - 6$

$= 12(\frac{1}{8}) + k(\frac{1}{4}) - \frac{1}{2} - 6$

$= \frac{3}{2} + \frac{k}{4} - \frac{1}{2} - \frac{12}{2}$

$0 = \frac{k}{4} + \frac{3-1-12}{2}$

$= \frac{k}{4} + \frac{-10}{2}$

$= \frac{k}{4} - 5$

$5 = \frac{k}{4}$

$\therefore k = 20$

p.177

$\rightarrow f(1)=10\ ;\ f(2)=51$

10. When $ax^3 - x^2 + 2x + b$ is divided by $x - 1$, the remainder is 10. When it is divided by $x - 2$, the remainder is 51. Find a and b.

$f(x)=ax^3-x^2+2x+b$

$f(1)=a(1)^3-(1)^2+2(1)+b$

$=a-1+2+b$

$10 = a+b+1$

$9 = a+b$

$9-a=b$

$f(2)=a(2)^3-(2)^2+2(2)+b$

$= 8a-4+4+b$

$51 = 8a+b$

$51=8a+(9-a)$

$51=8a+9-a$

$51-9=7a$

$42=7a$

$a=6$

$b=9-6$

$=3$

14. Show that $x - a$ is a factor of $x^4 - a^4$.

x^4-a^4

$=(x^2-a^2)(x^2+a^2)$

$=(x-a)(x+a)(x^2+a^2)$

p.177

16. Use the factor theorem to prove that $x^2 - x - 2$ is a factor of $x^3 - 6x^2 + 3x + 10$.

$f(x) = x^3 - 6x^2 + 3x + 10$

$f(-1) = (-1)^3 - 6(1)^2 + 3(-1) + 10$

$= -1 - 6 - 3 + 10$

$= 0$

$\therefore (x+1)$ is a factor

-1	1	-6	3	10
	↓	-1	7	-10
	1	-7	10	0R.

$x^2 - 7x + 10$

$= (x-5)(x-2)(x+1)$

$= (x-5)(x^2 - 1x - 2)$

17. Prove that $x + a$ is a factor of $(x + a)^5 + (x + c)^5 + (a - c)^5$.

$x + a = 0$

$x = -a$

$f(x) = (x+a)^5 + (x+c)^5 + (a-c)^5$

$f(-a) = (-a+a)^5 + (-a+c)^5 + (a-c)^5$

$= (-a+c)^5 + (a-c)^5$

$= (-1(a-c))^5 + (a-c)^5$

$= (-1)^5(a-c)^5 + (a-c)^5$

$= -(a-c)^5 + (a-c)^5$

$= 0$

$\therefore$ by factor theorem

$f(-a) = 0$

$\therefore x+a$ is a factor

3.7 Factoring a Sum or Difference of Cubes

Math Learning Target:
"I can factor fully a Sum **or** Difference of Cubes." $\Delta^3 \pm \square^3$

Sum of Cubes: $A^3 + B^3$

Difference of Cubes: $A^3 - B^3$

Ex.1 Apply the Factor Theorem to factor completely:

a) $y^3 + 8 = P(y)$

$P(-2) = (-2)^3 + 8$
$= -8 + 8$
$= 0$
$\because P(-2) = 0$
$\therefore y+2$ is a factor

-2	1	0	0	8
	↓	-2	4	-8
	1	-2	4	0 R

expected

$\therefore y^3 + 8 = (y+2)(y^2 - 2y + 4)$

$b^2 - 4ac$
$= (-2)^2 - 4(1)(4)$
$= 4 - 16$
$= -12$
$\because b^2 - 4ac < 0$
$\therefore$ No Real Solutions

b) $8x^3 - 27 = P(x)$

Hint:
$P(\frac{3}{2}) = 8(\frac{3}{2})^3 - 27$
$= 8(\frac{27}{8}) - 27$
$= 27 - 27$
$= 0$
$\because P(\frac{3}{2}) = 0$
$\therefore (x - \frac{3}{2})$ is a factor

3/2	8	0	0	-27
	↓	12	18	27
	8	12	18	0 R

$\therefore 8x^3 - 27 = (x - \frac{3}{2})(8x^2 + 12x + 18)$
$= (x - \frac{3}{2})2(4x^2 + 6x + 9)$
$= (2x - 3)(4x^2 + 6x + 9)$

$b^2 - 4ac$
$= (6)^2 - 4(4)(9)$
$= 36 - 144$
$= -108$
$\because b^2 - 4ac < 0$
$\therefore$ no real solutions.

c) $a^3 - b^3 = P(a)$

$P(b) = (b)^3 - b^3$
$= 0$
$\because P(b) = 0$
$\therefore a - b$ is a factor

b	1	0	0	$-b^3$
	↓	b	b^2	b^3
	1	b	b^2	0 R.

$\therefore a^3 - b^3 = (a-b)(a^2 + ba + b^2)$
$= (a-b)(a^2 + ab + b^2)$

Factor Formula for a Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor Formula for a Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Let's verify the result by expanding:

$$(a - b)(a^2 + ab + b^2)$$

$$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$$

$$= a^3 - b^3$$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Ex.2 Use the appropriate "new" formula to factor completely:

a) $8x^3 - 27$ (from the previous slide)

$a = 2x$ $b = 3$

$a^3 - b^3$

$(2x)^3 - (3)^3$

$= 8x^3 - 27$

$8x^3 - 27$

$= (2x - 3)((2x)^2 + (2x)(3) + (3)^2)$

$= (2x - 3)(4x^2 + 6x + 9)$

b) $27x^3 + 125y^3$

$= (3x)^3 + (5y)^3$

$= (3x + 5y)((3x)^2 - (3x)(5y) + (5y)^2)$

$= (3x + 5y)(9x^2 - 15xy + 25y^2)$

The Factor Theorem can be applied to any expression.
However, it ***may*** be more difficult to use than if one recognizes the expression as a sum/difference of cubes.
Hence, the following algorithm is suggested, from now on, when required to factor:

STOP Is the expression a sum/difference of cubes?
If so, use the appropriate formula.
Otherwise, apply the Factor Theorem directly.

Entertainment: p.182 #2acegi, 3, 4acegi, 5ac, 6

Are you factoring fully?