

Ex. Given $\triangle ABC$ with A (6, 15), B(-2, 3) and C(26, 3):

- a) Find M, the midpoint of AB.
- b) Find N, the midpoint of AC.
- c) Find the equation of the median, CM, from C.
- d) Find the equation of the median, BN, from B.
- e) Find the P.O.I. of the two medians.

$$\begin{aligned} a) M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= M \left(\frac{-2 + 6}{2}, \frac{3 + 15}{2} \right) \\ &= M \left(\frac{4}{2}, \frac{18}{2} \right) \\ &= M(2, 9) \end{aligned}$$

$$\begin{aligned} b) N &= \left(\frac{26 + 6}{2}, \frac{3 + 15}{2} \right) \\ &= N \left(\frac{32}{2}, \frac{18}{2} \right) \\ &= N(16, 9) \end{aligned}$$

$$\begin{aligned} c) m_{CM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 9}{26 - 2} \\ &= \frac{-6}{24} \\ &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{4}x + b \\ (9) &= -\frac{1}{4}(2) + b \\ 9 &= -\frac{1}{2} + b \\ 9 + \frac{1}{2} &= b \\ \frac{18}{2} + \frac{1}{2} &= b \\ \frac{19}{2} &= b \end{aligned}$$

$$\begin{aligned} \therefore y &= -\frac{1}{4}x + \frac{19}{2} \\ \hookrightarrow 4y &= -x + 38 \\ x + 4y &= 38 \quad (1) \\ -x + 3y &= 11 \end{aligned}$$

$$\begin{aligned} d) m_{BN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{9 - 3}{16 - (-2)} \\ &= \frac{6}{18} \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \therefore y &= \frac{1}{3}x + b \\ (9) &= \frac{1}{3}(16) + b \\ 9 &= \frac{16}{3} + b \\ \frac{27}{3} - \frac{16}{3} &= b \\ \frac{11}{3} &= b \end{aligned}$$

$$\begin{aligned} \therefore y &= \frac{1}{3}x + \frac{11}{3} \\ \hookrightarrow 3y &= x + 11 \\ x - 3y &= -11 \quad (2) \end{aligned}$$

$$\begin{aligned} \text{Sub in (1)} \\ x + 4(7) &= 38 \\ x &= 38 - 28 \\ &= 10 \end{aligned}$$

$\therefore (10, 7)$ is the P.O.I. of the two medians.

