

4.4 Graph $y = a(x - h)^2 + k$

Principles of Mathematics 10, pages 180–188

A

- Copy and complete the table for each parabola. Replace the heading for the second column with the equation for the parabola.

Property	$y = a(x - h)^2 + k$
Vertex	
Axis of symmetry	
Stretch or compression factor relative to $y = x^2$	
Direction of opening	
Values x may take	
Values y may take	

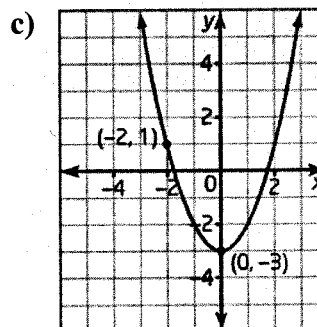
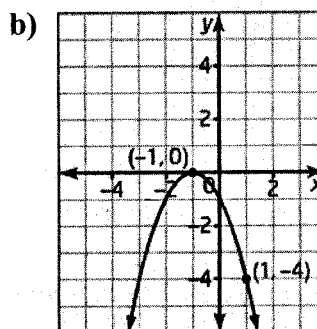
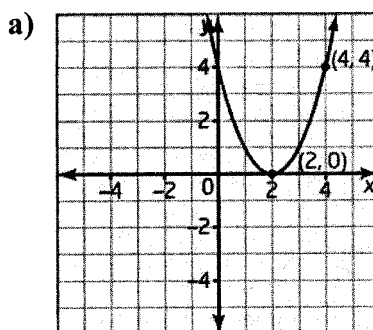
- $y = (x + 3)^2$
- $y = (x - 4)^2$
- $y = (x + 2)^2 + 5$
- $y = (x + 5)^2 - 3$
- $y = (x - 6)^2 + 7$
- $y = (x - 1)^2 - 8$
- $y = -(x + 8)^2 - 4$
- $y = 3(x + 7)^2 - 2$
- $y = -2(x + 3)^2 - 6$
- $y = -\frac{1}{2}(x + 5)^2 - 3$

- Use **Technology** Graph each parabola in question 1 using a graphing calculator.

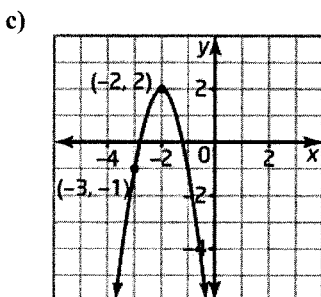
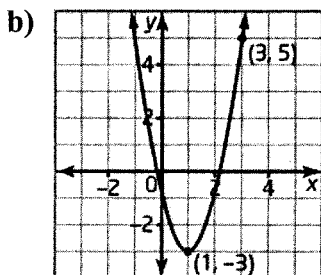
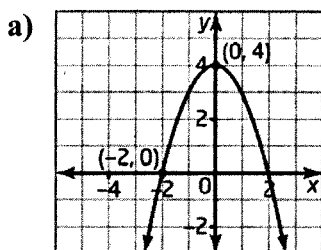
B

- Write an equation for the parabola with vertex $(3, 5)$, opening upward, and with no vertical stretch or compression.
- Write an equation for the parabola with vertex $(6, -2)$, opening downward, and with no vertical stretch or compression.

- Write an equation for the parabola with vertex $(-4, 5)$, opening downward, and with a vertical stretch of factor 3.
- Write an equation for the parabola with vertex $(-1, -7)$, opening upward, and with a vertical compression of factor 0.4.
- Write an equation for each parabola.



8. Write an equation for each parabola.



9. a) Find an equation for the parabola with vertex (2, 6) that passes through the point (5, 3).
- b) Find an equation for the parabola with vertex (-3, -4) that passes through the point (2, 6).
- c) Find an equation for the parabola with vertex (-1, 3) and x-intercept 1.
- d) Find an equation for the parabola with vertex (2, 5) and y-intercept -3.
- e) Find an equation for the parabola with vertex (-6, -2) that passes through the point (-3, -11).
- f) Find an equation for the parabola with vertex (6, 4) that passes through the point (8, 2).

C

10. The path of a football is modelled by the

relation $h = -\frac{1}{4}(d-12)^2 + 36$, where d is the horizontal distance, in metres, after it was kicked, and h is the height, in metres, above the ground.

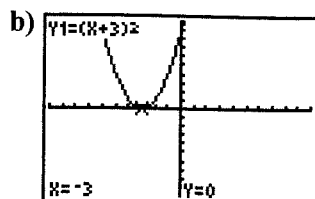
- a) Sketch the path of the football.
- b) What is the maximum height of the football?
- c) What is the horizontal distance when this occurs?
- d) What is the height of the football at a horizontal distance of 10 m?
- e) Find another horizontal distance where the height is the same as in part d).

11. A parabola has equation $y = 3(x+2)^2 + 4$. Write an equation for the parabola after each set of transformations.

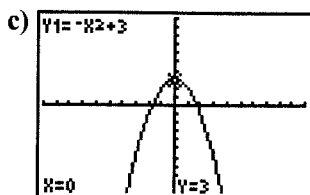
- a) a reflection in the x -axis
- b) a translation 6 units to the right
- c) a reflection in the x -axis, followed by a translation of 3 units downward
- d) a reflection in the y -axis

12. Find the equation for each of the following circles. Write your answer in the form $(x-h)^2 + (y-k)^2 = r^2$.

- a) radius 4, centred at (6, 0)
- b) radius 5, centred at (0, -2)
- c) radius 3, centred at (-7, 3)
- d) radius 6, centred at (-5, -4)



This graph is the graph of $y = x^2$ translated 3 units to the left.



This is the graph of $y = x^2$ reflected in the x -axis and then translated 3 units upward.

6. a) $y = x^2 + 5$

b) $y = x^2 - 3$

c) $y = x^2 + 6$

d) $y = x^2 - 8$

7. a) $y = (x + 4)^2$

b) $y = (x - 7)^2$

c) $y = (x + 6)^2$

d) $y = (x - 2)^2$

8. a) $y = 3x^2$

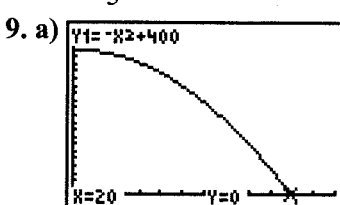
b) $y = \frac{1}{4}x^2$

c) $y = 5x^2$

d) $y = \frac{1}{6}x^2$

e) $y = -6x^2$

f) $y = -\frac{3}{5}x^2$



b) The y -intercept is 400. This represents the area of the backyard if there is no swimming pool. The x -intercept is 20. This represents the side length of the swimming pool, in metres, if the pool completely fills the backyard.

c) x must be greater than or equal to zero but less than or equal to 20

10. a) fourth diagram: 20 squares; fifth diagram: 30 squares.

b)

h	A	First Differences	Second Differences
1	2		
2	6	4	
3	12	6	2
4	20	8	2
5	30	10	2

Since the second differences are constant, the relation is a quadratic.

c) $y = x^2 + x$

d) Answers will vary.

4.4 Graph $y = a(x - h)^2 + k$, pages 41–42

1. a)

Property	$y = (x + 3)^2$
Vertex	$(-3, 0)$
Axis of symmetry	$x = -3$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 0$

b)

Property	$y = (x - 4)^2$
Vertex	$(4, 0)$
Axis of symmetry	$x = 4$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 0$

c)

Property	$y = (x + 2)^2 + 5$
Vertex	$(-2, 5)$
Axis of symmetry	$x = -2$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 5$

d)

Property	$y = (x + 5)^2 - 3$
Vertex	$(-5, -3)$
Axis of symmetry	$x = -5$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -3$

e)

Property	$y = (x - 6)^2 + 7$
Vertex	(6, 7)
Axis of symmetry	$x = 6$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 7$

f)

Property	$y = (x - 1)^2 - 8$
Vertex	(1, -8)
Axis of symmetry	$x = 1$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -8$

g)

Property	$y = -(x + 8)^2 - 4$
Vertex	(-8, -4)
Axis of symmetry	$x = -8$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -4$

h)

Property	$y = 3(x + 7)^2 - 2$
Vertex	(-7, -2)
Axis of symmetry	$x = -7$
Stretch or compression factor relative to $y = x^2$	3
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -2$

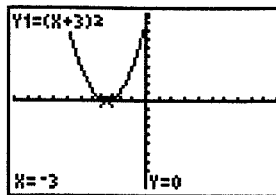
i)

Property	$y = -2(x + 3)^2 - 6$
Vertex	(-3, -6)
Axis of symmetry	$x = -3$
Stretch or compression factor relative to $y = x^2$	2
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -6$

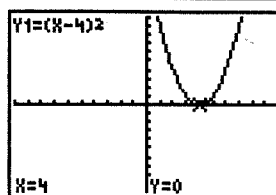
j)

Property	$y = -\frac{1}{2}(x + 5)^2 - 3$
Vertex	(-5, -3)
Axis of Symmetry	$x = -5$
Stretch or compression factor relative to $y = x^2$	$\frac{1}{2}$
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -3$

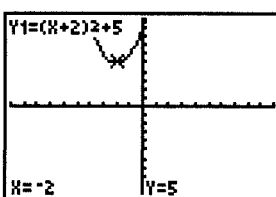
2. a)



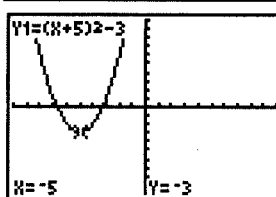
b)



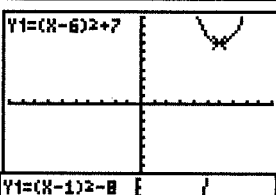
c)



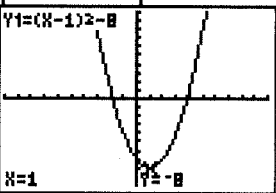
d)



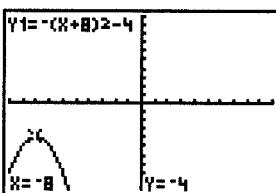
e)



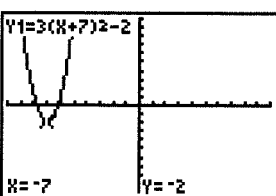
f)

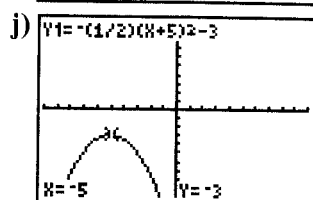
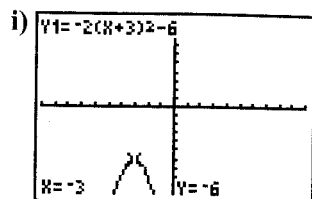


g)



h)





3. $y = (x-3)^2 + 5$

4. $y = -(x-6)^2 - 2$

5. $y = -3(x+4)^2 + 5$

6. $y = 0.4(x+1)^2 - 7$

7. a) $y = (x-2)^2$

b) $y = -(x+1)^2$

c) $y = x^2 - 3$

8. a) $y = -x^2 + 4$

b) $y = 2(x-1)^2 - 3$

c) $y = -3(x+2)^2 + 2$

9. a) $y = -\frac{1}{3}(x-2)^2 + 6$

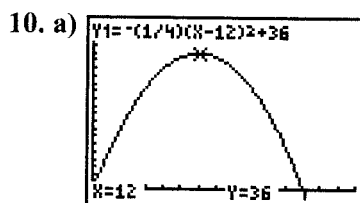
b) $y = \frac{2}{5}(x+3)^2 - 4$

c) $y = -\frac{3}{4}(x+1)^2 + 3$

d) $y = -2(x-2)^2 + 5$

e) $y = -(x+6)^2 - 2$

f) $y = -\frac{1}{2}(x-6)^2 + 4$



b) 36 m

c) 12 m

d) 35 m

e) 14 m

11. a) $y = -3(x+2)^2 - 4$

b) $y = 3(x-4)^2 + 4$

c) $y = -3(x+2)^2 - 7$

d) $y = 3(x-2)^2 + 4$

12. a) $(x-6)^2 + y^2 = 16$

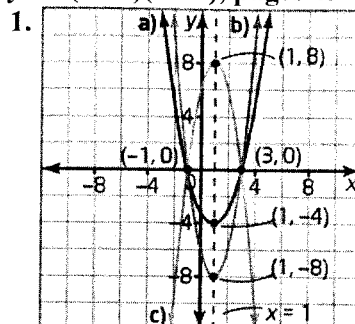
b) $x^2 + (y+2)^2 = 25$

c) $(x+7)^2 + (y-3)^2 = 9$

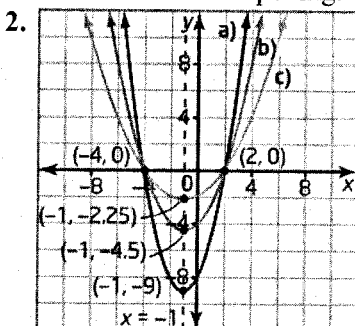
d) $(x+5)^2 + (y+4)^2 = 36$

4.5 Quadratic Relations of the Form

$y = a(x-r)(x-s)$, pages 43–44



The graphs all have the same x-intercepts and axis of symmetry. They differ in the vertical stretch of the parabola, the y-value of the vertex, and the direction of opening.



The graphs all have the same x-intercepts, axis of symmetry, and direction of opening. They differ in the vertical stretch of the parabola and the y-value of the vertex.

