6.1 Maximum/Minimum Problems   Date: _____

1. A large fountain located in a park has 35 water jets. One of the streams of water follows a parabolic path so that its height is given by the equation $h = -2x^2 + 6x + 5$ where h metres is the height of the stream of water at any distance x metres from its jet.
   What is the maximum distance the water jet can reach, rounded to the nearest tenth of a metre?   *(Answer: 9.5 m)*

2. Hermione's mother owns a manufacturing company that produces key rings. Last year, she collected data about the number of key rings produced per day and the corresponding profit. The data can be modelled by the relation $P = -2k^2 + 12k - 10$, where P is the profit in thousands of dollars and k is the number of key rings in thousands.
   a) How many key rings must be produced for the maximum profit?   *(Answer: 3000)*
   b) What is the maximum profit?   *(Answer: $8000)*
   c) Sketch this relation.   *(Answer: SEE @ BOTTOM OF PAGE)*

3. An architect has designed a modern building that is to be supported by a steel arch shaped like a parabola. This parabola can be modelled by the relation $y = -0.025x^2 + 2x$, where y represents the height of the arch and x represents the distance along the base, both in metres. What is the highest point on the parabolic arch?
   (Answer: 40 metres)

4. Hiroshi is trying out for the position of kicker on the football team. He wants to know at what angle he should kick the ball for maximum distance. He has used a machine that kicks footballs with constant velocity but at varying angles. Hiroshi has collected some data and used quadratic regression on his graphing calculator to determine that the relation between angle and distance is given by the equation $d = -0.1a^2 + 8.5a - 40$, where a is the angle in degrees, and d is the distance in metres.
   a) Determine the vertex of the parabola.   *(Answer: (42.5, 140.625))*
   b) Which angle gives the maximum distance?   *(Answer: 42.5 degrees)*
   c) For what values of a is the graph valid?   *(Answer: $5 < a < 80$)*

5. Herman arranged a game of water balloon volleyball for his daughter's birthday party. Suppose the balloon has a height of h metres, t seconds after it is tossed, as defined by $h = -5t^2 + 12t + 1$.
   When does it reach its maximum height?   *(Answer: 1.2 seconds)*

6. A field is bounded on one side by a river. The field is to be enclosed on three sides by a fence, to create a rectangular enclosure. The total length of fence to be used is 100 m. Use a quadratic model to determine the dimensions of the enclosure of maximum area.   *(Answer: 25 m by 50 m)*

Enrichment:

7. What is the maximum area of a triangle having 15 cm as the sum of its base and height?
   (Answer: 28.125 cm)

8. A research study has shown that 500 people attend a HHSS hockey game in a tournament when the admission price is $2. In the championship game, the price will be considered for an increase: for every 20¢ increase, 20 fewer people will attend. What price will maximize the revenue? What is the value of the maximum revenue?   *(Answer: $3.50; $1225)*

9. A rectangle has perimeter P.
   Find the maximum possible area of the rectangle.

9.   *(Answer: $P^2/16$ units^2)*

10. Find the minimum distance from (0, 0) to the line $3x + 2y - 12 = 0$.
    Round to the nearest hundredth.

10.   *(Answer: 3.33 units)*

FINAL ANSWER for #2c: