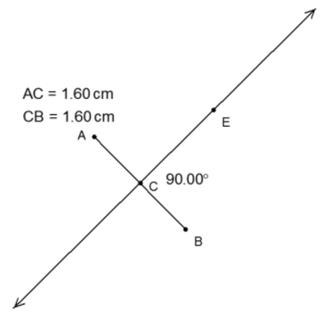
MPM 2DI Unit 3: Geometric Properties Summary

Note: *Each diagram is either a reduction or an enlargement of the original Geometer's Sketchpad diagram. Hence, each linear measurement stated relates to the* ***original*** *linear measurement.*

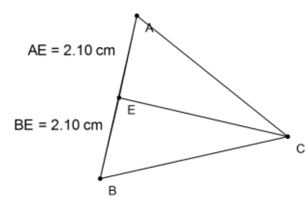
To **bisect** means to divide in two equal parts. A **right bisector** is a line that bisects a line segment at a 90° angle. For example, the line through C is a right bisector:



When points are **collinear**, they belong to the same line or line segment. In the above diagram, C and E are collinear; points A, C and B are also collinear. Whenever two or more line segments or lines meet at a single point, they are **concurrent.**

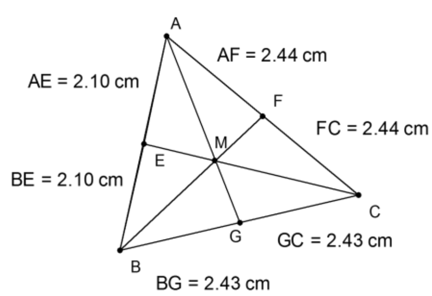
TRIANGLES

The **median** of a triangle is a line segment that joins a vertex of a triangle to the midpoint of the opposite side. For example, line segment CE is one median:



Don't forget that each median bisects the area of the triangle too! Hence, area Δ AEC = area Δ BEC in the above triangle.

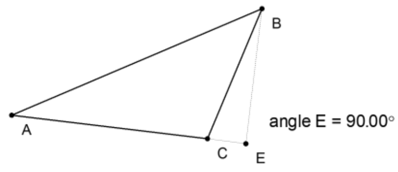
The **centroid** of a triangle is the point where all three medians of a triangle meet. For example, M is the centroid in Δ ABC:



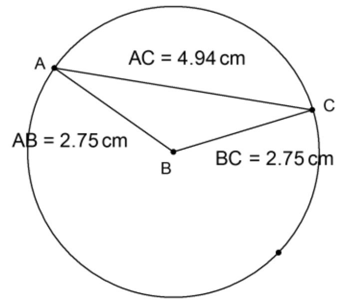
Did you know that the centroid always divides each median into two parts where one length is twice that of the other length? In the above diagram, BM is twice as long as MF. Hence, BM : MF = 2 : 1

An **altitude** (or **height**) of a triangle is the shortest line segment between a vertex and its opposite side. ***Note:***

1. Since it is the shortest line segment, it must be perpendicular to its opposite side;
2. The opposite side may need to be extended for the altitude to exist (see below).

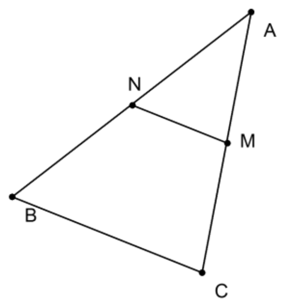


To prove that a triangle is **isosceles**, one must prove either ***exactly*** two of its sides are equal OR ***exactly*** two of its angles are the same. If it is not isosceles, it must be **equilateral** or **scalene**. Consider Δ ABC inscribed in the circle below. We know it is isosceles because two sides make use of the equal radii of the circle, and AC is not the same length as the radii:



In any isosceles triangle, the median from the vertex between its equal sides bisects the angle at the vertex. **(In the previous diagram, sketch this median!)** The same median will also be an altitude to that vertex.

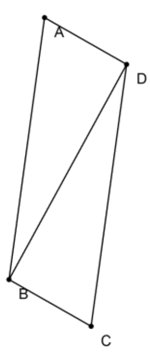
The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and half its length. In the example below, M and N are the midpoints of AC and AB respectively. This means MN is parallel to BC and is half the length of BC, that is, MN ∥ BC and BC:MN = 2:1.



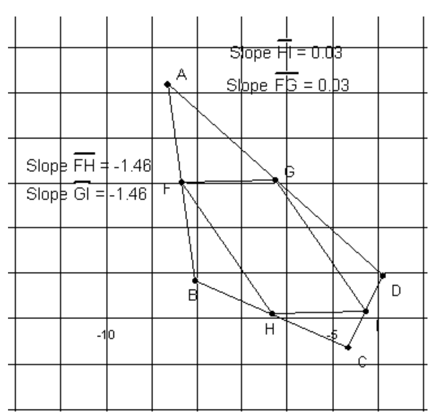
QUADRILATERALS

A **quadrilateral** is a polygon with four sides. A **parallelogram** is a quadrilateral with opposite sides that are parallel to each other. In a parallelogram, the opposite sides are also always equal in length, and the opposite interior angles are the same. A **rhombus** is a parallelogram where all sides are equal in length.

The **diagonals** of a parallelogram always bisect each other. In the example below, the diagonal AC would bisect diagonal BD. In other words, AC would cut BD in two equal lengths. **(Sketch this below!)**



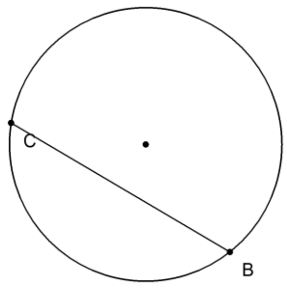
In any quadrilateral, joining the midpoints of adjacent sides will form a parallelogram. For example, FGHI is a parallelogram in the diagram below.



This parallelogram is called a **Varignon parallelogram**. If the original quadrilateral is a square, the Varignon parallelogram is a square too! If the original quadrilateral is a rhombus, the Varignon parallelogram is a rectangle. If the original quadrilateral is a rectangle, the Varignon parallelogram is a rhombus.

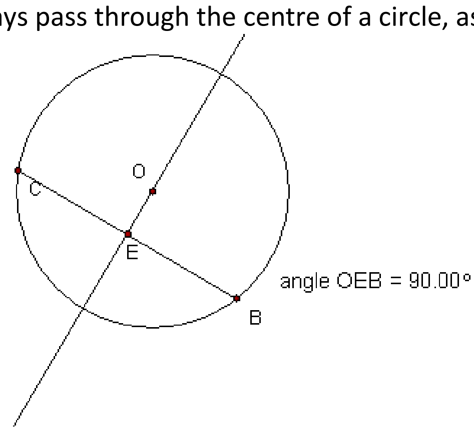
CIRCLES

A **chord** of a circle is a line segment that joins two points on the circumference of a circle.
In the circle below, BC is a chord:

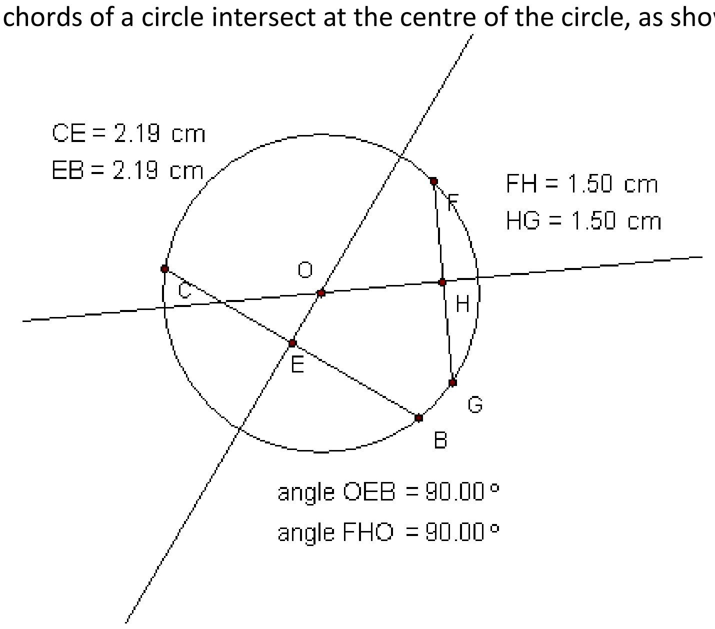


Note: the diameter of a circle is also a chord!

The right bisector of a chord will always pass through the centre of a circle, as shown below.



The right bisectors of two chords of a circle intersect at the centre of the circle, as shown below.



For any three points that are NOT collinear, there is only ONE circle that will pass through all three points!