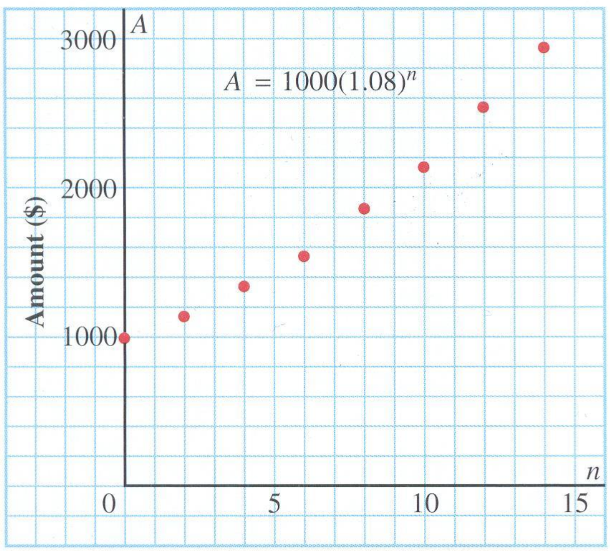
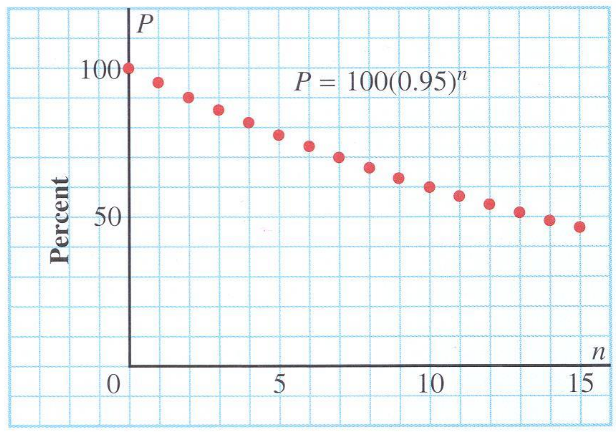
7.1 Introduction to Exponential Functions

Exponents were originally defined as a notation for repeated multiplication. Repeated multiplication occurs frequently in applications involving growth and decay.

Compound Interest

Suppose you make a long-term investment of \$1000 at a fixed interest rate of 8% compounded annually. The amount, A dollars, of your investment after n years is represented by the equation $A = 1000(1.08)^n$. In this equation, n is a natural number because it indicates the number of years.



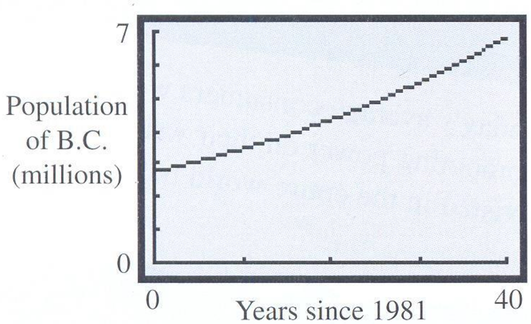


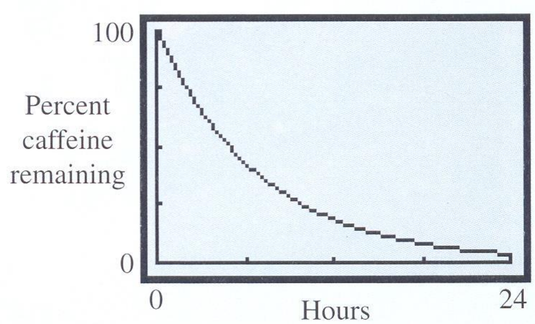
Light Transmission through Glass

Several layers of glass are stacked together. Each layer reduces the light passing through it by 5%. The percent, P, of light passing through n layers is represented by the equation $P = 100(0.95)^n$. Again, n is a natural number because it indicates the number of layers of glass.

Growth of British Columbia's Population

The population, P million, of British Columbia can be modelled by the equation $P = 2.76(1.022)^n$, where n is the number of years since 1981.





How Long Caffeine Stays in Your Bloodstream

Coffee, tea, cola, and chocolate contain caffeine. When you consume caffeine, the percent, P, left in your body can be modelled as a function of the elapsed time, n hours, by the equation $P = 100(0.87)^n$.

Comparing the Equations

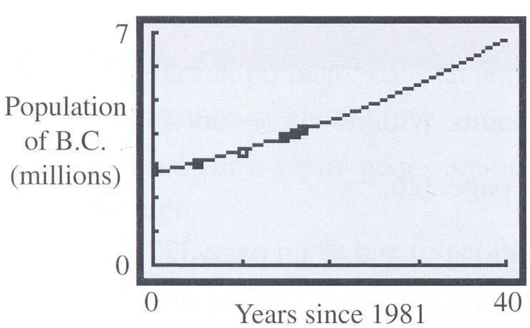
The situations on page 326 and above are examples of exponential growth and exponential decay. Consider the similarities in the equations of the functions.

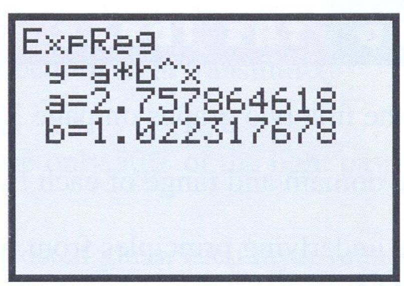
Exponential Growth		Exponential Decay	
$A = 1000(1.08)^n$	①	$P = 100(0.95)^n$	②
Initial value	Growth factor (greater than 1)	Initial value	Decay factor (less than 1)
$P = 2.76(1.022)^n$	③	$P = 100(0.87)^n$	④

In each equation, the variable in the expression on the right side appears in an exponent. Functions whose defining equations have this property are exponential functions.

> An *exponential function* has an equation that can be written in the form $y = Ab^x$, where A and b are constants, and $b > 0$.

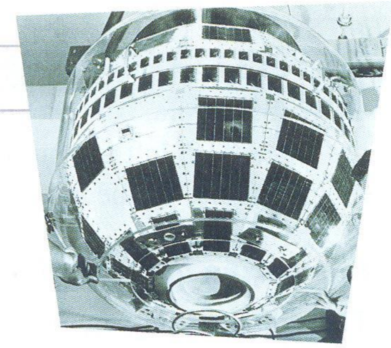
In some applications of exponential functions, there is an underlying principle from which the equation is derived. Equations ① and ② above were obtained in this way. In other applications, the equation is found using empirical data. Equations ③ and ④ were determined by calculating the equation of the exponential curve of best fit, see pages 332 and 333. The screens below show the plotted points and the calculated results used to determine equation ③. The second screen shows that the equation of the exponential curve of best fit is $y \doteq 2.76(1.022)^x$.





Equations determined in this way will be provided in many examples and exercises in this chapter. One of these equations occurs in the *Example*.

Example



The first artificial satellites were put in orbit in the late 1950s. The cumulative mass, c tonnes, of all the satellites in orbit can be modelled as a function of n, the number of years since 1960, by the equation $c = 120 \times 1.12^n$.

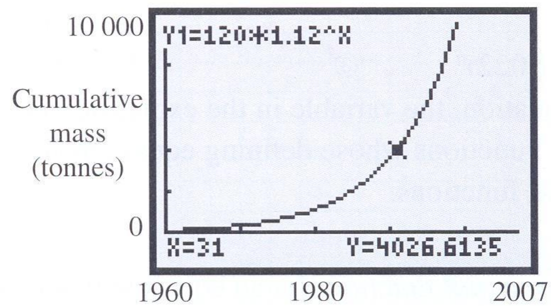
a) Graph the equation.

b) Estimate the cumulative mass in 1969, the year when a person first walked on the moon.

c) According to the model, when did the cumulative mass become 4000 t?

Solution

a) Use appropriate window settings:
$0 \le x \le 47; -1000 \le y \le 10\,000$



b) Substitute 9 for n in the equation.
$c = 120 \times 1.12^9$
$\doteq 333$
In 1969, the cumulative mass was about 333 t.

c) Substitute 4000 for c in the equation.
$4000 = 120 \times 1.12^n$
Trace to the point where the y-coordinate is closest to 4000. This occurs when the x-coordinate is 31. According to the model, the mass became 4000 t in 1960 + 31, or 1991.

In the *Example* part c, we solved the equation $1.20 \times 1.12^n = 4000$ by graphing and tracing. This is an example of an *exponential equation*. In the next two sections, we will develop a more efficient method to solve exponential equations.

DISCUSSING THE IDEAS

1. Explain why the first two graphs on page 326 show points, with no curves drawn.

2. Determine the domain and range of each function on page 326.

3. a) Explain the underlying principles from which equations ① and ② on page 327 were obtained.

b) Do you think there would be an underlying principle for obtaining equation ③ or equation ④? Explain.