Example 1

Consider the family of quadratic functions, where each function has zeros –2 and 3.

a) Write the equation of the family, then state two functions that belong to this family.

b) Determine the equation of the member of the family that passes through (1, 4).

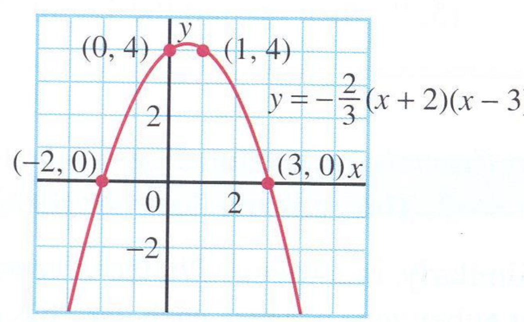
c) Graph the function in part b.

Solution

a) Since the zeros are –2 and 3, the equation for this family of functions is $y = a(x + 2)(x - 3)$. Two other functions are found by substituting any real numbers for a.
For $a = -4$, $y = -4(x + 2)(x - 3)$
For $a = 5$, $y = 5(x + 2)(x - 3)$

b) Since the graph passes through (1, 4), the coordinates of this point satisfy the equation.
Substitute $x = 1$ and $y = 4$ in $y = a(x + 2)(x - 3)$.
$4 = a(1 + 2)(1 - 3)$
$4 = a(3)(-2)$
$a = -\frac{2}{3}$
The equation of the function is $y = -\frac{2}{3}(x + 2)(x - 3)$.

c) The graph is concave down. Its axis of symmetry is halfway between $x = -2$ and $x = 3$; that is, $x = 0.5$. By symmetry, the point (0, 4) also lies on the graph. Draw a smooth curve through all known points.



When the equation of a polynomial function is expressed in factored form, we can use the factors to graph the function.

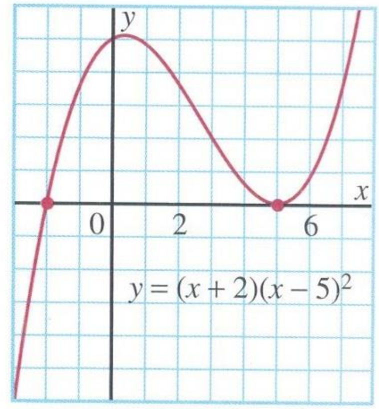
Example 2

Sketch a graph of each function.

a) $y = (x + 2)(x - 5)^2$ b) $y = (x + 2)^2(x - 5)^2$

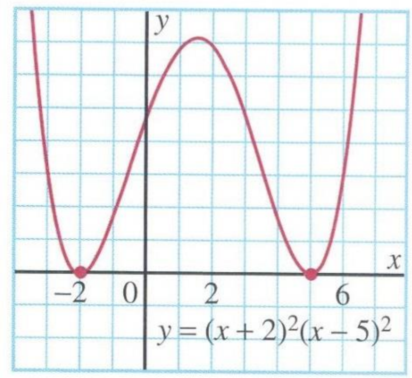
Solution

a) $y = (x + 2)(x - 5)^2$



This cubic function has a zero at $x = -2$ and a zero of order 2 at $x = 5$. Mark these points on the x-axis. Since the coefficient of x^3 is positive, the graph extends from the 3rd quadrant to the 1st quadrant. Begin in the 3rd quadrant. The curve passes up through (–2, 0), back down to touch the x-axis at (5, 0), then up into the 1st quadrant.

b) $y = (x + 2)^2(x - 5)^2$



This quartic function has zeros of order 2 at $x = -2$ and $x = 5$. Since the coefficient of x^4 is positive, the graph extends from the 2nd quadrant to the 1st quadrant. Begin in the 2nd quadrant. The curve passes down to touch the x-axis at (–2, 0), up into the 2nd quadrant and 1st quadrant, back down to touch the x-axis at (5, 0), then up into the 1st quadrant.

In *Example 2a*, the function $y = (x + 2)(x - 5)^2$ changes sign at the zero $x = -2$. This function does not change sign at $x = 5$, which is a zero of order 2.

Similarly, in *Example 2b*, the function $y = (x + 2)^2(x - 5)^2$ does not change sign at either zero because each zero has order 2.

In *Example 2*, the polynomial functions were given in factored form. You already know how to factor polynomials of degree 2. In Chapter 5, you will learn how to factor polynomials of degree greater than 2.

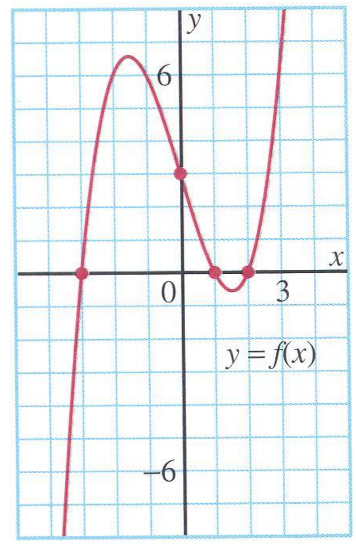
Many polynomial functions do not factor. We can use a graphing calculator to graph such functions, then use Trace or Calculate to approximate each zero.

We may use the idea of a family of functions to determine the equation of a graph of a polynomial function.

Example 3

Determine the equation of the polynomial function below right.

Solution



From the shape of the graph, the function is cubic.
Its equation has the form $y = a(x - b)(x - c)(x - d)$.
From the graph, the zeros of the function are –3, 1, and 2.
So, the equation is $y = a(x + 3)(x - 1)(x - 2)$.
From the graph, the y-intercept is 3.
Substitute $x = 0$, $y = 3$ in $y = a(x + 3)(x - 1)(x - 2)$.

$$3 = a(3)(-1)(-2)$$
$$3 = 6a$$
$$a = \frac{3}{6}$$
$$= \frac{1}{2}$$

The equation is $y = \frac{1}{2}(x + 3)(x - 1)(x - 2)$.

DISCUSSING THE IDEAS

In the solution of *Example 1*, why did we not expand the product $(x + 2)(x - 3)$ in the equation of the function?